15-2

Personal Math Trainer

15.2 Isosceles and Equilateral Triangles - Class & Homework

HMH

Notes

1 Find the angle measure. $m \angle P = \boxed{54}^\circ$
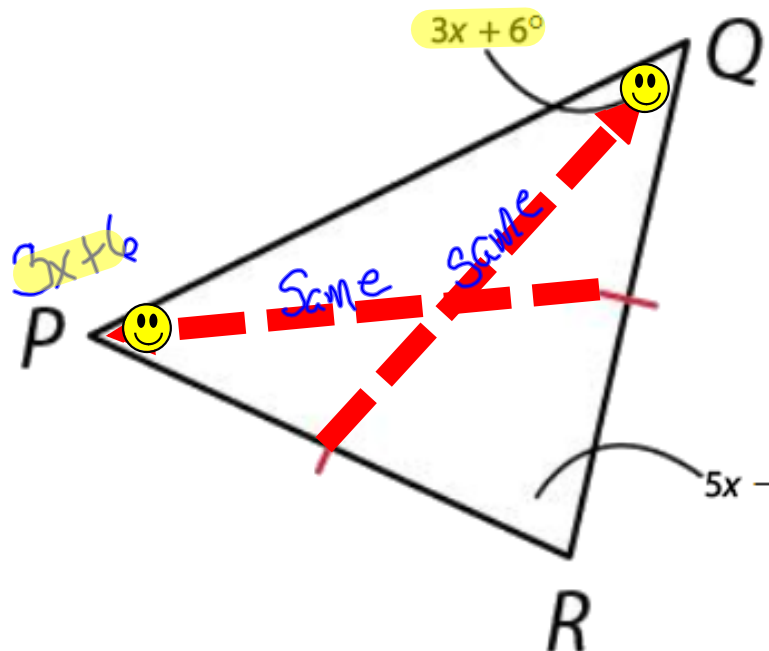
$3x + 6^\circ$

$3x + 6$

Same   Same

P

Q

R

$5x -$

*To find the measure of the angle of the triangle, first find the value of x using the Triangle Sum Theorem.

$$m \angle P + m \angle Q + m \angle R = 180^\circ$$
$$2(3x + 6) + (5x - 8) = 180$$
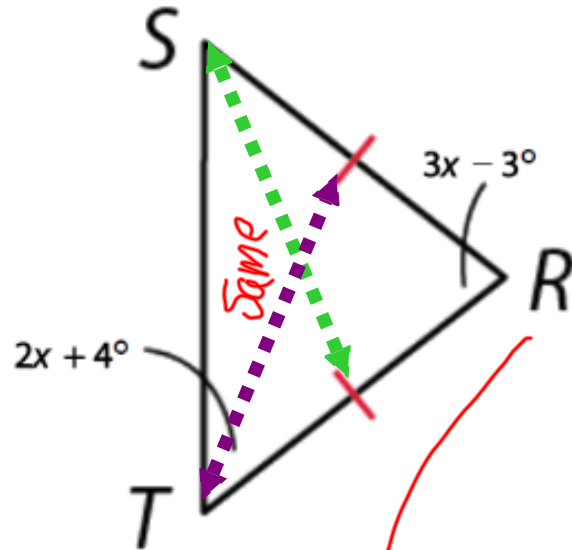$$6x + 12 + 5x - 8 = 180$$
$$11x + 4 = 180$$
$$11x = 176$$
$$x = 16$$

So, $m \angle P = (3x + 6)^\circ$
$$= (3(16) + 6)^\circ$$
$$= 54^\circ.$$

2 Find the measure of the indicated angle

$m \angle R = \boxed{72}\ °$

*To find the measure of the angle of the triangle, first find the value of x using the Triangle Sum Theorem.

$$m \angle S = m \angle T = (2x + 4)°$$

$$m \angle R + m \angle S + m \angle T = 180^c$$
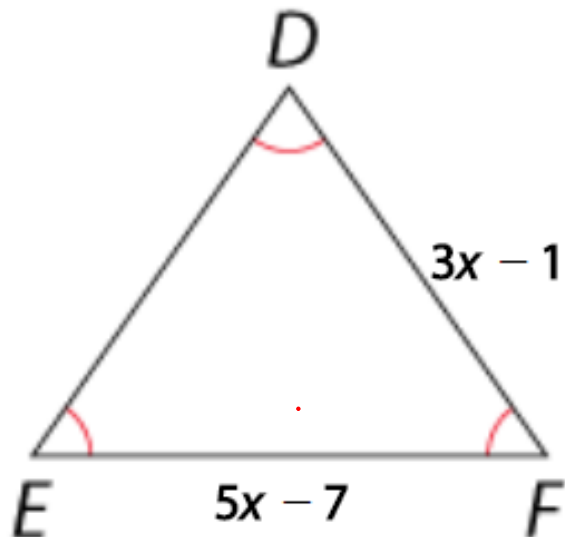
$$(3x - 3) + 2(2x + 4) = 180$$

$$3x - 3 + 4x + 8 = 180$$

$$7x + 5 = 180$$

$$7x = 175$$

$$x = 25$$

Triangle labels: S, R, T with $3x - 3°$ at R, $2x + 4°$ at T, "Same"

So, $m \angle R = (3x - 3)°$
$= (3(25) - 3)°$
$= 72°.$

3

3 Find the length of the indicated side. **The length of side $\overline{DE}$ is** $\boxed{8}$.

D

$3x - 1$

$5x - 7$

E   F

Substitute 3 for x.

To find the length of each side of the triangle, first find the value of x. Use the Converse of the Equilateral Triangle Theorem:

Definition of congruence

$$\overline{DF} \cong \overline{EF}$$

$$DF = EF$$

$$3x - 1 = 5x - 7$$
$$-5x + 1 \quad -5x + 1$$
$$\overline{\quad -2x \quad = \quad -6 \quad}$$

$$-2x = -6$$

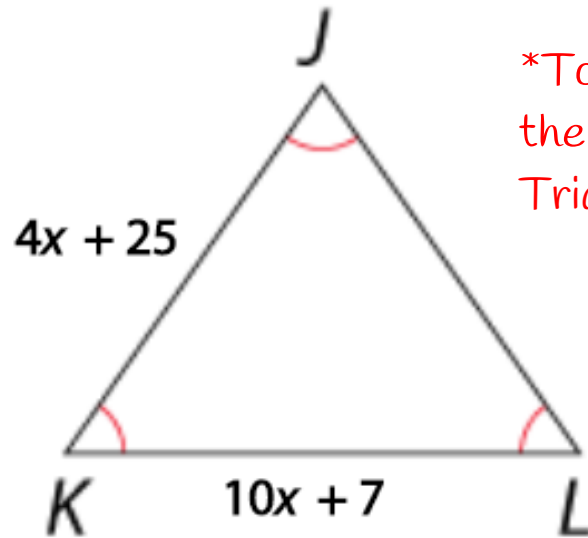$$x = 3$$

$$DE = EF$$
$$DE = 5(3) - 7$$
$$DE = 8$$

4 Find the length of the indicated side.   **The length of side $\overline{KL}$ is** [37] .

J

4x + 25

K   10x + 7   L

*To find the length of each side of the triangle, first find the value of x. Use the Converse of the Equilateral Triangle Theorem:
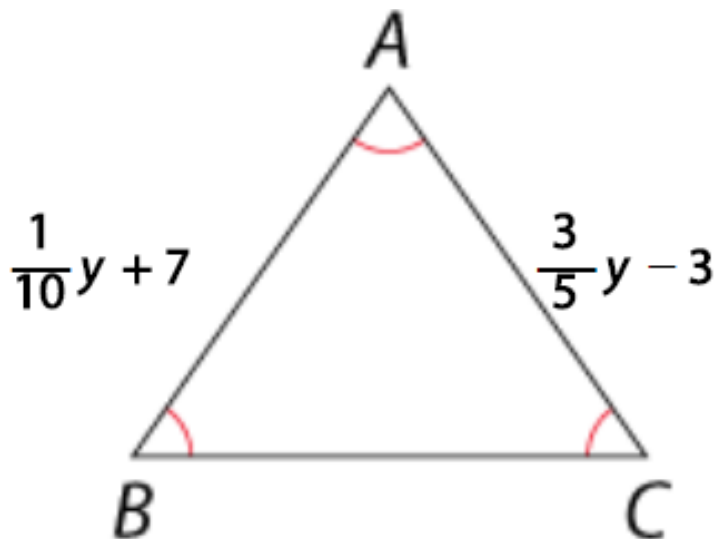
Definition of congruence

$$JK = KL$$

$$4x + 25 = 10x + 7$$
$$-10x - 25 \qquad -10x - 25$$

$$-6x = -18$$

$$x = 3$$

Substitute 3 for x.

$$KL = 10x + 7$$
$$KL = 10(3) + 7$$
$$KL = 37$$

5 Katie's tutorial service is going so well that she is having shirts made with the equilateral triangle emblem. She has given the t-shirt company these dimensions. What is the length of each side of the triangle in centimeters?
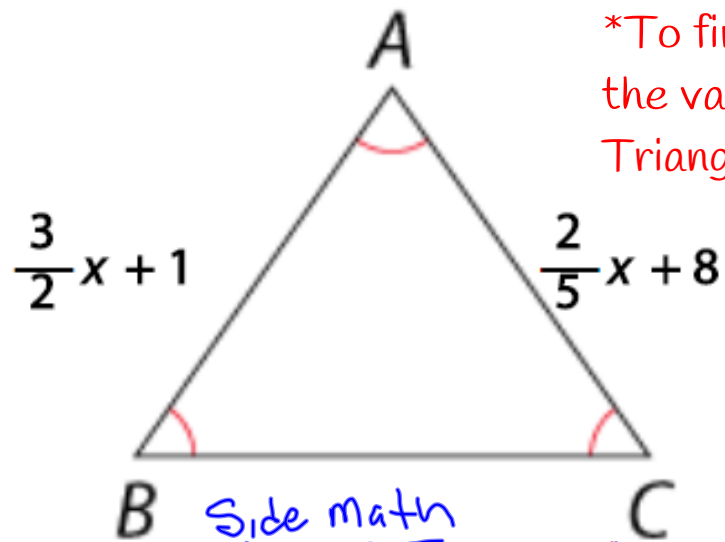
The length of each side of the triangle is [9] centimeters.

A

$\frac{1}{10}y + 7$ $\frac{3}{5}y - 3$

B   C

$AB = AC$

$$\frac{1}{10}y + \cancel{7}_{-7} = \frac{3}{5}y - 3_{-7}$$

do common denominator

$$\frac{1}{10}y = \frac{3}{5}\cancel{y} - 10$$

$$-\frac{3}{5}y \qquad -\frac{3}{5}\cancel{y}$$

$$-\frac{1}{2}y = -10$$

$$y = 20$$

Substitute 20 for y into $\frac{1}{10}y + 7$.

$$\frac{1}{10}(20) + 7 = 9$$

6

6 Find the length of the indicated side. **The length of side $\overline{AB}$ is** $\boxed{\frac{116}{11}}$.

To find the length of each side of the triangle, first find the value of x. Use the Converse of the Equilateral Triangle Theorem:

A

$\frac{3}{2}x + 1$ $\frac{2}{5}x + 8$

B   Side math   C

$$AB = AC$$

$$\frac{3}{2}x + 1 = \frac{2}{5}x + 8$$
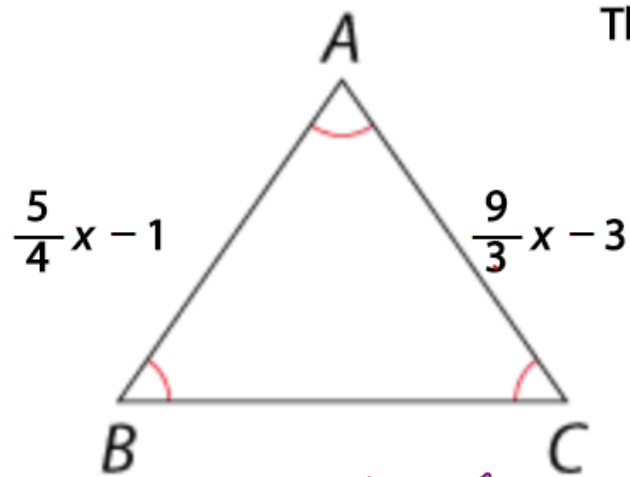$$\quad -1 \qquad\qquad -1$$
$$\frac{3}{2}x = \frac{2}{5}x + 7$$
$$-\frac{2}{5}x \qquad -\frac{2}{5}x$$

$$\frac{10}{11}\cdot\frac{11}{10}x = 7\frac{10}{11}$$

$$x = \frac{70}{11}$$

Side math

$$\frac{5}{5}\cdot\frac{3}{2} - \frac{2}{5}\cdot\frac{2}{2}$$

$$\frac{15}{10} - \frac{4}{10} = \frac{11}{10}$$

Substitute $\frac{70}{11}$ for x. $AB = \frac{3}{2}x + 1 = \frac{3}{2}\left(\frac{70}{11}\right) + 1 = \frac{105}{11} + \frac{1}{1}\cdot\frac{11}{11} = \frac{116}{11}$

7

7 Find the length of the indicated side.

The length of side $\overline{BC}$ is ☐ .

$\dfrac{5}{4}x - 1$

$\dfrac{9}{3}x - 3$

$\overline{AB} \cong \overline{AC}$

$AB = AC$

$\dfrac{5}{4}x - 1 = \dfrac{9}{3}x - 3$

$+1 \qquad +1$

$\dfrac{5}{4}x = \dfrac{9}{3}x - 2$

$-\dfrac{9}{3}x$

$-\dfrac{9}{3}x$

$\dfrac{7}{?} - \dfrac{1}{4}x = -2 \cdot -\dfrac{4}{7}$

$x = \dfrac{8}{7}$

Side math

$\dfrac{3}{3} \cdot \dfrac{5}{4} - \dfrac{9}{3} \cdot \dfrac{4}{4}$

$\dfrac{15}{12} - \dfrac{36}{12} = -\dfrac{21}{12} \div \dfrac{3}{3} = -\dfrac{7}{4}$

Substitute $\dfrac{8}{7}$ for x. $BC = \dfrac{5}{4}x - 1$

$BC = \dfrac{5}{4}\left(\dfrac{8}{7}\right) - 1$

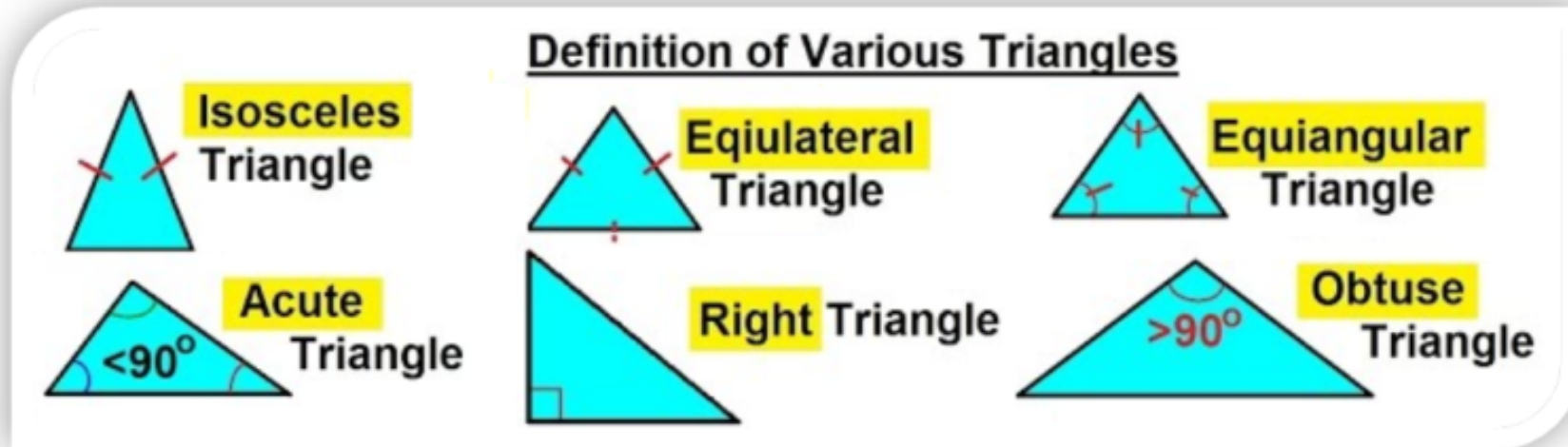$BC = \dfrac{10}{7} - \dfrac{7}{7} = \dfrac{3}{7}$

8

8 Given $\triangle JKL$ with $m \angle J = 55°$ and $m \angle L = 70°$.

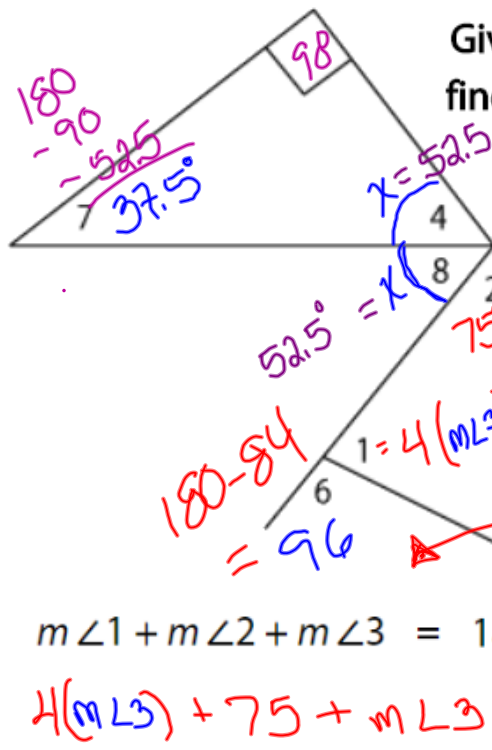Is the triangle an acute, isosceles, obtuse, or right triangle?

Find the missing angle:

$$m \angle K = 180 - 55 - 70 = 55°$$

$m \angle K =$ [55] °, so the triangle is an [acute ▼] triangle because all angle measures are [less than ▼] 90°.

Definition of Various Triangles

Isosceles Triangle

Eqiulateral Triangle

Equiangular Triangle

Acute Triangle <90°

Right Triangle

Obtuse Triangle >90°

9 Use the figure to answer the following.

Given $m\angle 2 = 75°$, $m\angle 1 = 4 \cdot m\angle 3$, and $\angle 4 \cong \angle 8$,

find $m\angle 1$, $m\angle 3$, $m\angle 4$, $m\angle 5$, $m\angle 6$, $m\angle 7$, and $m\angle 8$.

98

180
-90
-52.5
$37.5°$

$x = 52.5$

$52.5 = x$

$180 - 84$
$1 = 4(m\angle 3) = 84$
$= 96$

$84 \qquad = \boxed{21}°, m\angle 4 = \boxed{52.5}°,$
$159 \qquad = \boxed{96}°, m\angle 7 = \boxed{37.5}°,$

52.5

$2x + 75 = 180$
$\quad -75 \quad\quad -75$
$\quad \dfrac{2x}{2} \quad = \dfrac{105}{2}$
$x = 52.5$

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

$4(m\angle 3) + 75 + m\angle 3 = 180$

$5(m\angle 3) + 75 = 180$
$\qquad -75 \qquad -75$
$\dfrac{5(m\angle 3)}{5} = \dfrac{105}{5}$

$m\angle 3 = 21$

$21°$

180
-21
$159°$

Continue ▶

Use properties of angles and triangles to determine the measure of each angle.

$$m \angle 1 + m \angle 2 + m \angle 3 = 180°$$

Find $m \angle 3$.

$$4 \cdot m \angle 3 + 75° + m \angle 3 = 180°$$

$$m \angle 3 = 21°$$

Find $m \angle 1$.

$$m \angle 1 = 4 \cdot m \angle 3$$

$$m \angle 1 = 4 \cdot 21°$$

$$m \angle 1 = 84°$$

Find $m \angle 5$.

$$m \angle 3 + m \angle 5 = 180°$$

$$21° + m \angle 5 = 180°$$

$$m \angle 5 = 159°$$

$$m \angle 1 + m \angle 6 = 180°$$

$$84° + m \angle 6 = 180°$$

$$m \angle 6 = 96°$$

$$m \angle 2 + m \angle 4 + m \angle 8 = 180°$$

$$75° + m \angle 4 + m \angle 4 = 180°$$

$$m \angle 4 = 52.5°$$

Find $m \angle 8$.

$$m \angle 8 = m \angle 4$$

$$m \angle 8 = 52.5°$$

Find $m \angle 7$.

$$m \angle 7 + m \angle 8 + 90° = 180°$$

$$m \angle 7 + 52.5° + 90° = 180°$$

$$m \angle 7 = 37.5°$$

9 Continue

10 What is

$$|-2x+7| = 25$$

$+$ $-$

$$-2x+7 = 25$$ $$-2x+7 = -25$$

$$-2x+7 = 25$$ $$-2x+7 = -25$$

$$-2x+7-7 = 25-7$$ $$-2x+7-7 = -25-7$$

$$-2x = 18$$ $$-2x = -32$$

$$\frac{-2x}{-2} = \frac{18}{-2}$$ $$\frac{-2x}{-2} = \frac{-32}{-2}$$

$$x = -9 \checkmark$$ $$x = 16 \checkmark$$

✓ Step 1: Isolate the **absolute value** expression.
✓ Step 2: Set the quantity inside the **absolute value** notation equal to + and −.
✓ Step 3: Solve for the unknown in both **equations**.

The only way
to learn
mathematics
is to do
mathematics.

PAUL HALMOS