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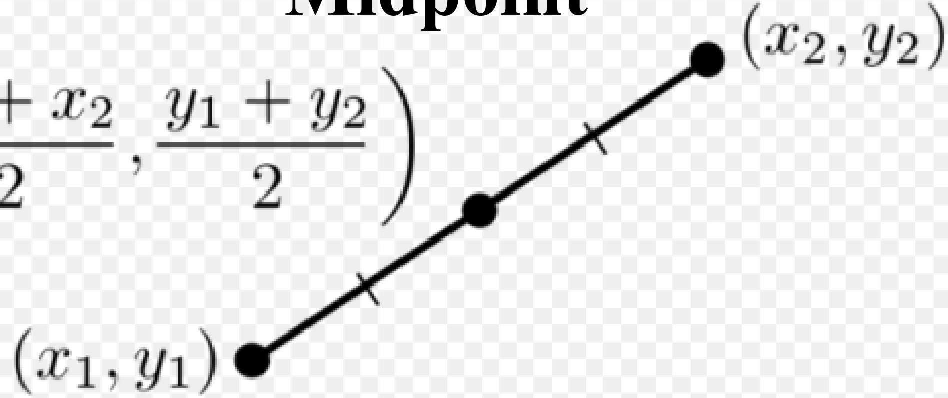
16.1 Segment Length and Midpoints

16.1

Midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

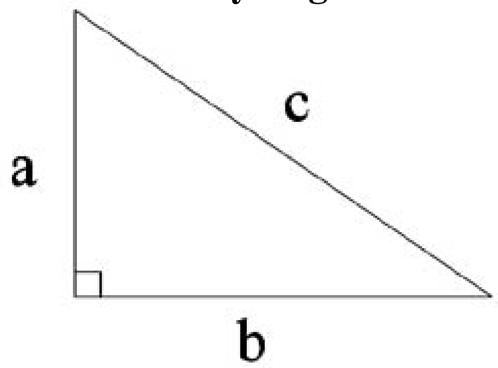


The Distance Formula

(x_1, y_1) (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Pythagorean theorem formula



$$a^2 + b^2 = c^2$$

Enter the midpoint of the segment whose endpoints are (6, 7) and (−8, 14).

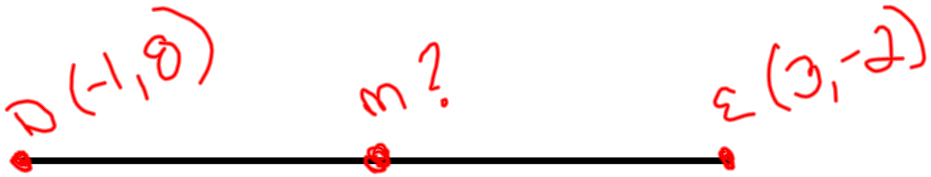
$x_1 \quad y_1 \qquad x_2 \quad y_2$

$$\left(\frac{6-8}{2}, \frac{7+14}{2}\right)$$

$$\left(\frac{-2}{2}, \frac{21}{2}\right)$$

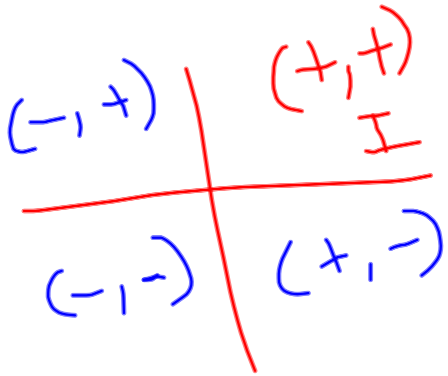
$$\left(-1, \frac{21}{2}\right)$$

Show that the statement is true. If $\overline{DE}$ has endpoints $D(-1, 8)$ and $E(3, -2)$, then the midpoint M of $\overline{DE}$ lies in Quadrant I.



$$\left(\frac{-1+3}{2}, \frac{8-2}{2}\right)$$

$$\left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$$



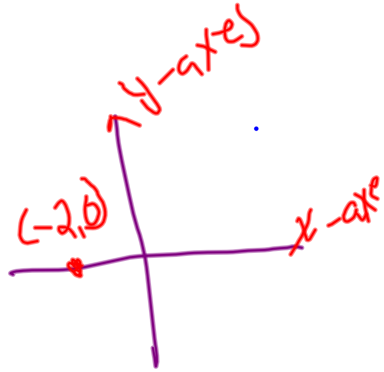
The midpoint M is (1 , 3). M lies in Quadrant I, since the x-coordinate is positive and the y-coordinate is positive.

Show that the statement is true. If $\overline{ST}$ has endpoints $S(-4, -5)$ and $T(0, 5)$, then the midpoint M of $\overline{ST}$ lies on the x-axis.

3

$$\left(\frac{-4+0}{2}, \frac{-5+5}{2}\right)$$

$$\left(\frac{-4}{2}, \frac{0}{2}\right) = (-2, 0)$$



The midpoint M is (-2, 0). M lies on the x-axis, since the y-coordinate is 0.

Show that the statement is true. If $\overline{GH}$ has endpoints $G(-2, 1)$ and $H(4, -1)$, then the midpoint M of $\overline{GH}$ lies on the line $y = -x + 1$.

4

$\left(\frac{-2+4}{2}, \frac{1-1}{2}\right) = \left(\frac{2}{2}, \frac{0}{2}\right) = (1, 0)$
x , y

Method -1 : plug & check

$y = -x + 1$
$0 \mid -1 + 1$
$0 \mid 0$ ✓

It is on the line

Method-2

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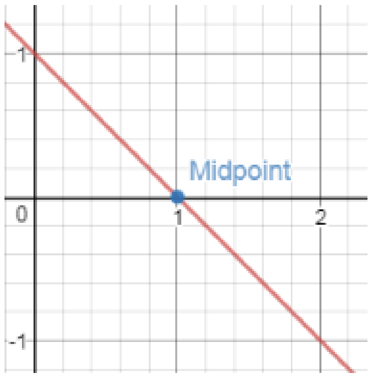
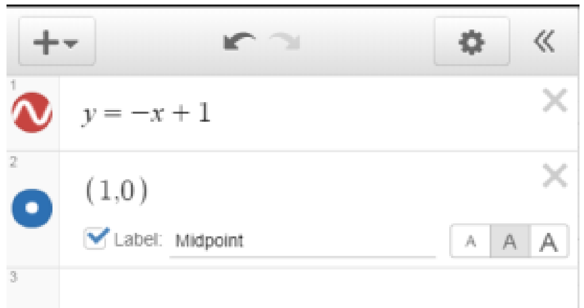




The midpoint M is (1, 0). The midpoint lies on the line $y = -x + 1$ since its coordinates satisfy the equation.

The sign shows distances from a rest stop to the exits for different towns along a straight section of highway. The state department of transportation is planning to build a new exit to Freestone at the midpoint of the exits for Roseville and Edgewood. When the new exit is built, what will be the distance from the exit for Midtown to the exit for Freestone?

5

$\frac{35+57}{2} = \frac{92}{2} = 46$ miles

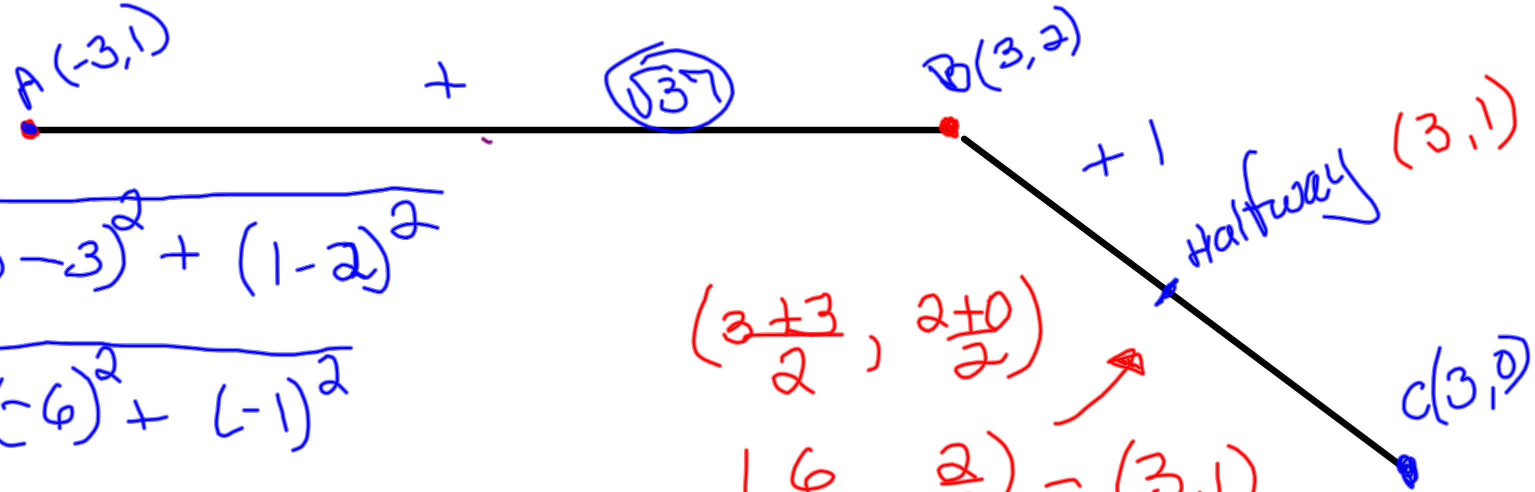
Midtown to Freestone

$46 - 17 = 29$ miles

When the new exit is built the distance from the exit for Midtown to the exit for Freestone will be 29 miles.

On a town map, each unit of the coordinate plane represents 1 mile. Three branches of a bank are located at $A(-3, 1)$, $B(3, 2)$, and $C(3, 0)$. A bank employee drives from Branch A to Branch B and then drives halfway to Branch C before getting stuck in traffic. What is the minimum total distance the employee may have driven before getting stuck in traffic? Round to the nearest tenth of a mile if necessary.

6



$\sqrt{(-3-3)^2 + (1-2)^2}$

$\sqrt{(-6)^2 + (-1)^2}$

$\sqrt{36+1} = \sqrt{37}$

$\left(\frac{3+3}{2}, \frac{2+0}{2}\right)$

$\left(\frac{6}{2}, \frac{2}{2}\right) = (3,1)$

$\sqrt{(3-3)^2 + (2-1)^2}$

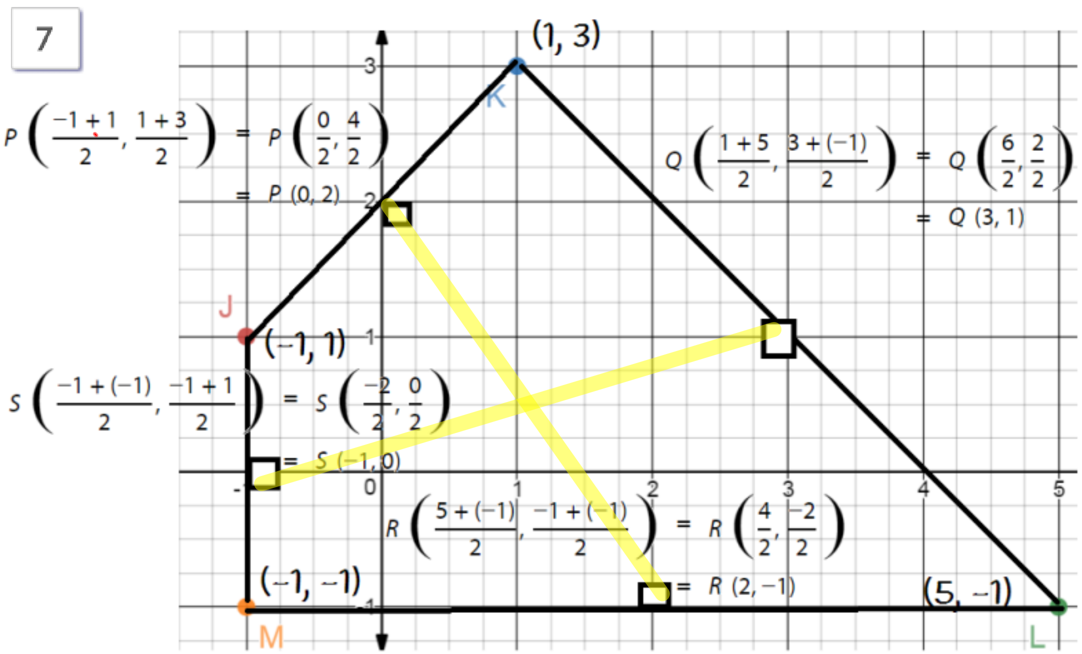
$\sqrt{0^2 + 1^2} = 1$

Total: $\sqrt{37} + 1$

$\approx 6.08 + 1$

≈ 7.1

A city planner designs a park that is a quadrilateral with vertices at $J(-1, 1)$, $K(1, 3)$, $L(5, -1)$, and $M(-1, -1)$. There is an entrance to the park at the midpoint of each side of the park. A straight path connects each entrance to the entrance on the opposite side. Assuming each unit of the coordinate plane represents 10 meters, what is the total length of the paths to the nearest meter? Round your answer to the nearest whole number



Find the path length of $\overline{PR}$.

$$
\begin{aligned}
PR &= \sqrt{(0-2)^2 + (2-(-1))^2} \\
&= \sqrt{-2^2 + 3^2} \\
&= \sqrt{13}
\end{aligned}
$$

Find the path length of $\overline{SQ}$.

$$
\begin{aligned}
SQ &= \sqrt{(3-(-1))^2 + (1-0)^2} \\
&= \sqrt{(4)^2 + (1)^2} \\
&= \sqrt{17}
\end{aligned}
$$

The total length of the paths is $\sqrt{13} + \sqrt{17} \approx 7.73$.

Multiply the length by 10 meters since each unit on the coordinate plane represents 10 meters.

The total length of the paths is approximately 77 meters.

Point M is the midpoint of $\overline{AB}$. The coordinates of point A are (−8, 1) and the coordinates of M are (−3, 1). What are the coordinates of point B?

8

Let (x, y) be the coordinates of point B.

Then by the Midpoint Formula, solve for x.

$$-3 = \frac{-8 + x}{2}$$
$$-6 = -8 + x$$
$$2 = x$$

Solve for y.

$$1 = \frac{1 + y}{2}$$
$$2 = 1 + y$$
$$1 = y$$

The coordinates of point B are (2, 1).

9

Use a definition, postulate, or theorem to find the value desired.

Point M is the midpoint between points A and B. If $A(-9, 2)$ and $B(-3, -4)$ find the location of M.

Use the midpoint formula.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-9 + (-3)}{2}, \frac{2 + (-4)}{2}\right)$$

$$= M(-6, -1)$$

M is located at the point $(-6, -1)$.

10

Show that the statement is true. If $\overline{AB}$ has endpoints $A(3, -6)$ and $B(-3, 6)$, then the midpoint M of $\overline{AB}$ is the origin.

Use the midpoint formula to find the midpoint M.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{3 + (-3)}{2}, \frac{-6 + 6}{2}\right)$$

$$= M\left(\frac{0}{2}, \frac{0}{2}\right)$$

$$= M(0, 0)$$

The midpoint M is (0 , 0), which is ▼ the origin.