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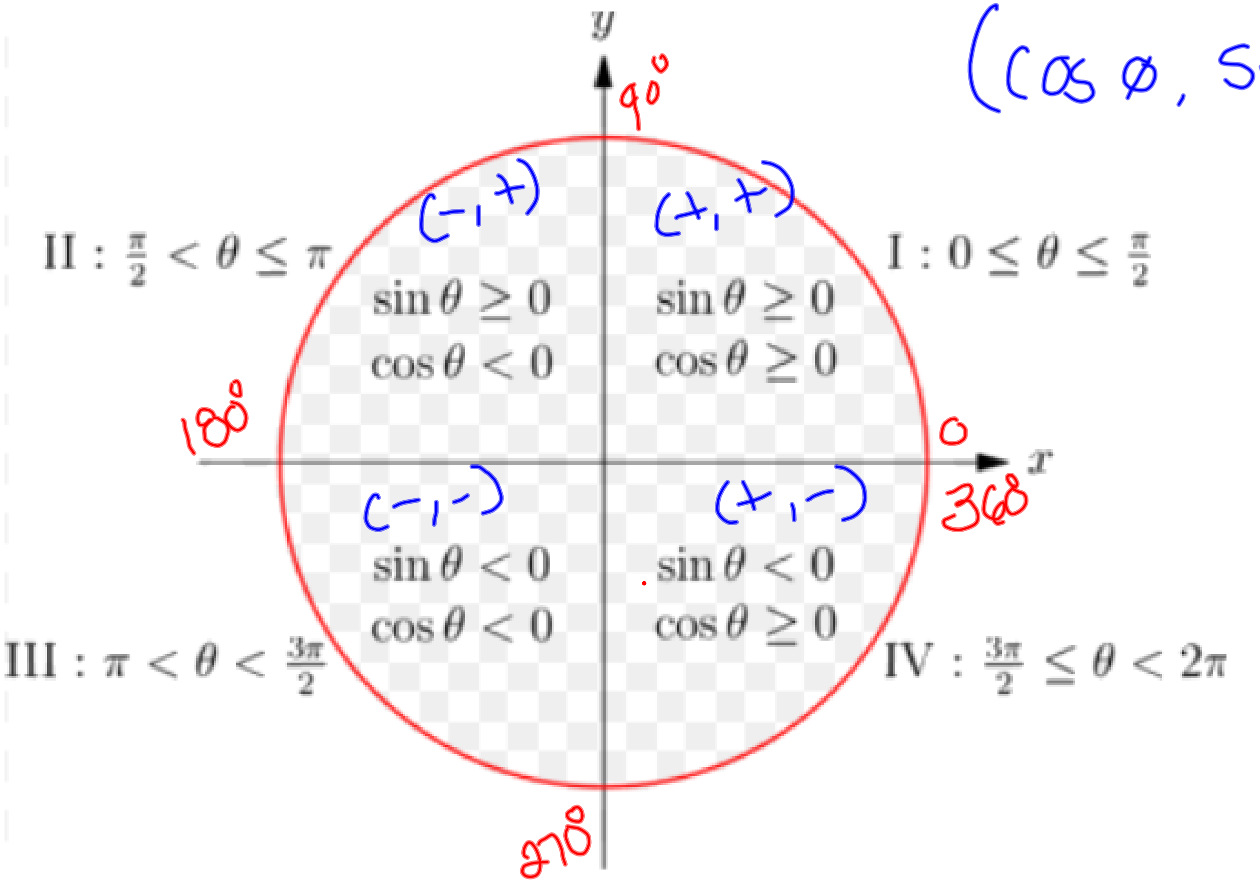
18.5 Using a Pythagorean Identity - Class & Homework

18.5

(cos ø, sin ø)



Pythagorean Identity

$$\cos^2\theta + \sin^2\theta = 1$$

Simplify and enter the trigonometric expression in terms of cos θ.

$$\frac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$$

Pythagorean Identity

$$\sin^2\theta = 1 - \cos^2\theta$$

Factor: $(x^2 - 1) = (x+1)(x-1)$

$$\frac{\sin^2\theta}{1-\cos\theta} = \frac{(1-\cos^2\theta)}{1-\cos\theta} = \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$

$$= 1 + \cos\theta$$

2 Given that $\cos\theta = -0.996$ where $\frac{\pi}{2} < \theta < \pi$, find $\sin\theta$. Then find $\tan\theta$. If necessary, round to 3 decimal places.

Since θ lies in Quadrant II, where $\sin\theta > 0$, $\sin\theta \approx$ 0.089.

$\tan\theta \approx$ -0.089

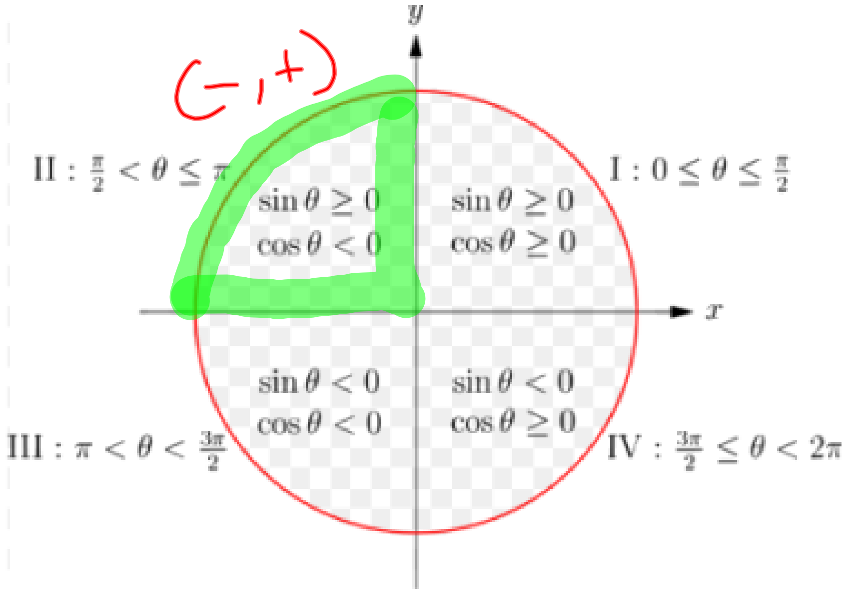
Use the identity to solve for $\sin\theta$.

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Substitute for $\cos\theta$.

$$= \pm\sqrt{1-(-0.996)^2} \approx 0.089.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \approx \frac{0.089}{-0.996} \approx -0.089$$



3 Find the approximate value of each trigonometric function.

Given that $\cos\theta = 0.197$ where $\frac{3\pi}{2} < \theta < 2\pi$, find $\sin\theta$. If necessary, round to 3 decimal places.

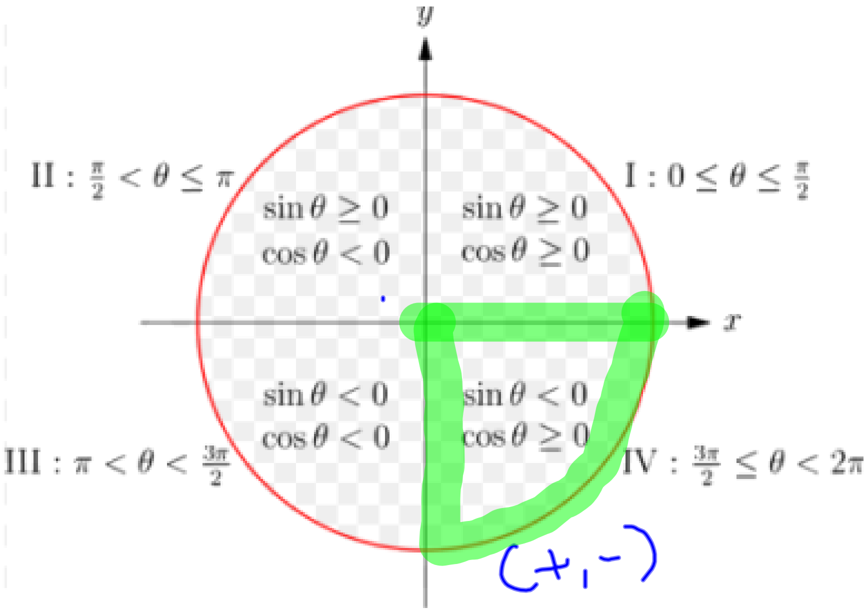
Since θ lies in Quadrant IV, where $\sin\theta < 0$, $\sin\theta$ is approximately -0.98 .

Use the identity to solve for $\sin\theta$.

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Substitute for $\cos\theta$.

$$= \pm\sqrt{1-(0.197)^2} = -0.98.$$



4 Find the approximate value of each trigonometric function.

Given that $\cos\theta = -0.482$ where $\pi < \theta < \frac{3\pi}{2}$, find $\sin\theta$. If necessary, round to 3 decimal places.

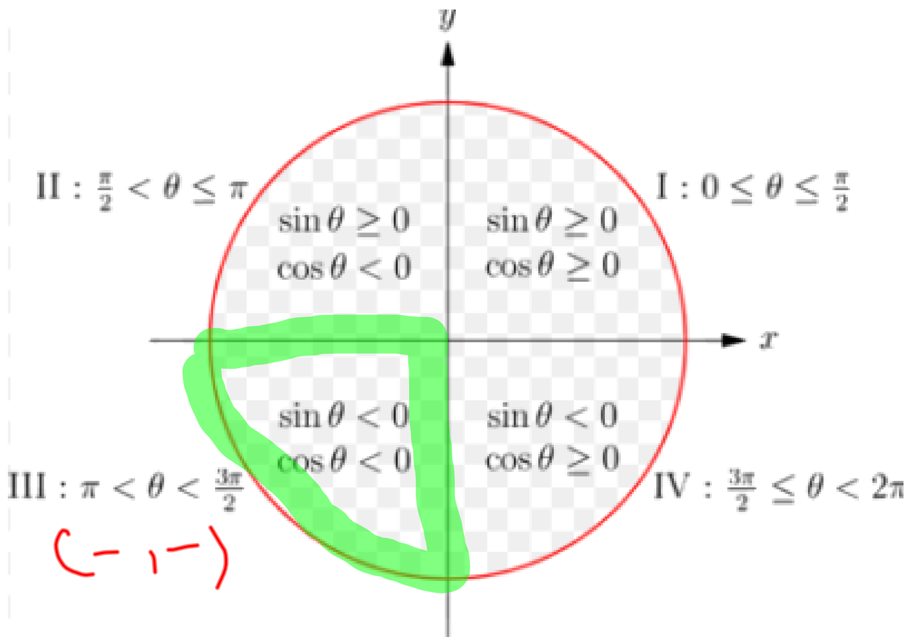
Since θ lies in Quadrant III, where $\sin\theta < 0$, $\sin\theta$ is approximately -0.876.

Use the identity to solve for $\sin\theta$.

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$$

Substitute for $\cos\theta$.

$$= \pm\sqrt{1 - (-0.482)^2} \approx -0.876.$$



5

Find the approximate value of each trigonometric function.

Given that $\sin\theta = -0.644$ where $\pi < \theta < \frac{3\pi}{2}$, find $\cos\theta$. If necessary, round to 3 decimal places.

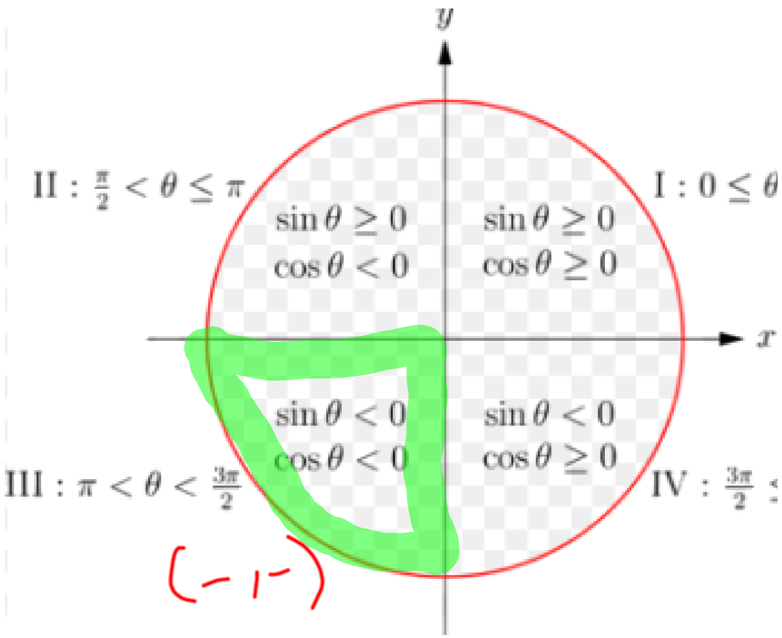
Since θ lies in Quadrant III, where $\cos\theta < 0$, $\cos\theta$ is approximately -0.765 .

Use the identity to solve for $\cos\theta$.

$$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

Substitute for $\sin\theta$.

$$= \pm\sqrt{1 - (-0.644)^2} \approx -0.765.$$



6

Find the approximate value of each trigonometric function.

Given that $\sin\theta = 0.513$ where $0 < \theta < \frac{\pi}{2}$, find $\cos\theta$. If necessary, round to 3 decimal places.

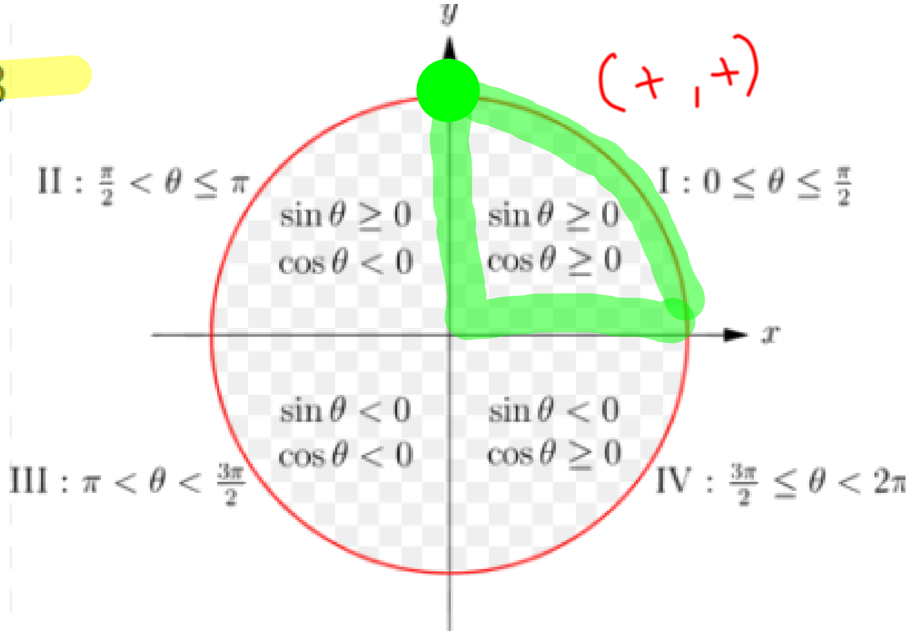
Since θ lies in Quadrant I, where $\cos\theta > 0$, $\cos\theta$ is approximately 0.858.

Use the identity to solve for $\cos\theta$.

$$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

Substitute for $\sin\theta$.

$$= \pm\sqrt{1 - (0.513)^2} \approx 0.8$$



7 Find the approximate value of each trigonometric function.

Given that $\sin\theta = -0.444$ where $\frac{3\pi}{2} < \theta < 2\pi$, find $\cos\theta$. If necessary, round to 3 decimal places.

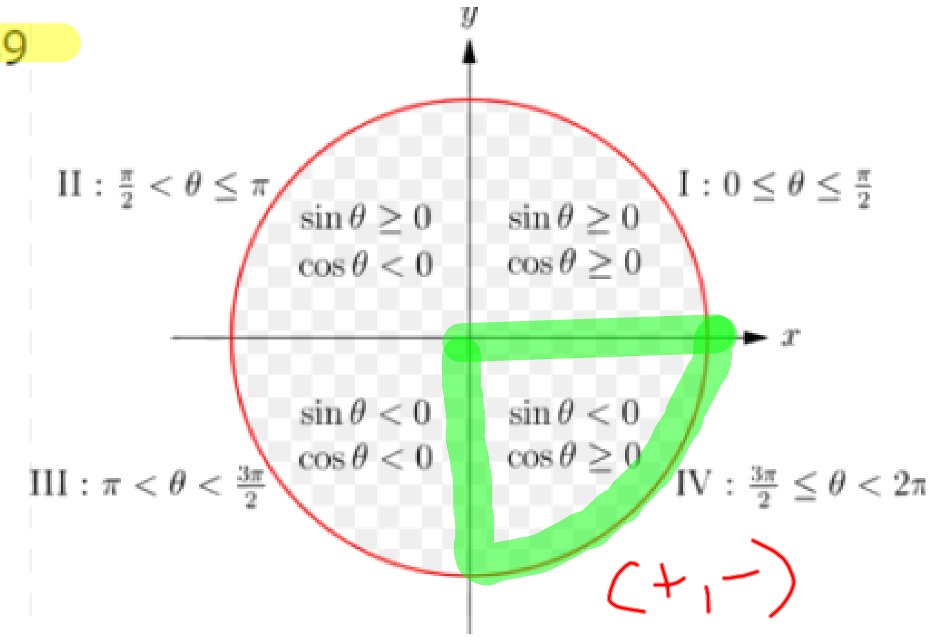
Since θ lies in Quadrant IV, where $\cos\theta > 0$, $\cos\theta$ is approximately 0.896 .

Use the identity to solve for $\cos\theta$.

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Substitute for $\sin\theta$.

$$= \pm\sqrt{1-(-0.444)^2} \approx 0.89$$



8 Find the approximate value of each trigonometric function.

Given that $\tan\theta \approx -9.357$ where $\frac{\pi}{2} < \theta < \pi$, find the values of $\sin\theta$ and $\cos\theta$. If necessary, round to 3 decimal places.

In Quadrant II, $\sin\theta \approx$ 0.992 and $\cos\theta \approx$ -0.106.

First, write $\sin\theta$ in terms of $\cos\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos\theta\tan\theta = \frac{\sin\theta}{\cos\theta}\cos\theta$$

$$\sin\theta = \cos\theta\tan\theta$$

Substitute the value $\tan\theta$.

$$\sin\theta \approx -9.357\cos\theta$$

Use the Pythagorean Identity.

$$\sin^2\theta + \cos^2\theta = 1$$

Substitute for $\sin\theta$.

$$(-9.357\cos\theta)^2 + \cos^2\theta \approx 1$$

Square.

$$87.5534\cos^2\theta + \cos^2\theta \approx 1$$

Combine like terms.

$$88.5534\cos^2\theta \approx 1$$

Solve for $\cos^2\theta$.

$$\cos^2\theta \approx 0.0113$$

Solve for $\cos\theta$.

$$\cos\theta \approx -0.106$$

In Quadrant II $\cos\theta$ is negative.

$$\cos\theta \approx -0.106$$

$$\sin\theta \approx -9.357\cos\theta \approx 0.992$$

9 Find the approximate value of each trigonometric function.

Given that $\tan\theta \approx 5.637$ where $\pi < \theta < \frac{3\pi}{2}$, find the values of $\sin\theta$ and $\cos\theta$. If necessary, round to 3 decimal places.

In Quadrant III, $\sin\theta \approx$ -0.992 and $\cos\theta \approx$ -0.176.

First, write $\sin\theta$ in terms of $\cos\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos\theta\tan\theta = \frac{\sin\theta}{\cos\theta}\cos\theta$$

$$\sin\theta = \cos\theta\tan\theta$$

Substitute the value $\tan\theta$.

$$\sin\theta \approx 5.637\cos\theta$$

Use the Pythagorean Identity.

$$\sin^2\theta + \cos^2\theta = 1$$

Substitute for $\sin\theta$.

$$(5.637\cos\theta)^2 + \cos^2\theta \approx 1$$

$$31.776\cos^2\theta + \cos^2\theta \approx 1$$

$$32.776\cos^2\theta \approx 1$$

$$\cos^2\theta \approx 0.031$$

$$\cos\theta \approx \pm 0.176$$

$$\cos\theta \approx -0.176$$

$$\sin\theta \approx 5.637\cos\theta \approx -0.992$$

10 Find the approximate value of each trigonometric function.

Given that $\tan\theta \approx -1.231$ where $\frac{3\pi}{2} < \theta < 2\pi$, find the values of $\sin\theta$ and $\cos\theta$. If necessary, round to 3 decimal places.

In Quadrant IV, $\sin\theta \approx$ -0.777 and $\cos\theta \approx$ 0.631.

First, write $\sin\theta$ in terms of $\cos\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos\theta\tan\theta = \frac{\sin\theta}{\cancel{\cos\theta}}\cancel{\cos\theta}$$

$$\sin\theta = \cos\theta\tan\theta$$

Substitute the value $\tan\theta$.

$$\sin\theta \approx -1.231\cos\theta$$

Use the Pythagorean Identity.

$$\sin^2\theta + \cos^2\theta = 1$$
$$(-1.231\cos\theta)^2 + \cos^2\theta \approx 1$$
$$1.515\cos^2\theta + \cos^2\theta \approx 1$$
$$2.515\cos^2\theta \approx 1$$
$$\cos^2\theta \approx 0.398$$
$$\cos\theta \approx \pm 0.631$$

$$\cos\theta \approx 0.631$$
$$\sin\theta \approx -1.231\cos\theta \approx -0.777$$


Never say,
"I can't"
Always say,
"I'll try"