

# ADD RATIONAL EXPRESSIONS

## Learning Objective

We will add rational expressions<sup>1</sup>.

### CFU

What are we going to learn?

What does *expressions* mean?  
*Expressions* means \_\_\_\_\_.

## Activate Prior Knowledge

The **least common denominator (LCD)** is the **lowest multiple shared** by a set of denominators.

- *The least common denominator can be found by using each factor the greatest number of times it occurs in a single fraction.*

Find the least common denominator.

$$1. \quad \frac{x+7}{x-5} \text{ and } \frac{x+5}{x-2}$$
$$(x-5)(x-2)$$

$$2. \quad \frac{x+3}{x-1} \text{ and } \frac{x+5}{x-4}$$
$$(x-1)(x-4)$$

Create equivalent fractions.

$$3. \quad \frac{x+3}{x-1} = \frac{x^2 - x - 12}{(x-1)(x-4)}$$
$$\frac{x+3}{x-1} \cdot \frac{x-4}{x-4}$$
$$\frac{x^2 - x - 12}{(x-1)(x-4)}$$

$$4. \quad \frac{x+5}{x-4} = \frac{x^2 + 4x - 5}{(x-1)(x-4)}$$
$$\frac{x+5}{x-4} \cdot \frac{x-1}{x-1}$$
$$\frac{x^2 + 4x - 5}{(x-1)(x-4)}$$

### Make Connection

Students, you already know how to find the least common denominator and create equivalent fractions. Now, we are going to find least common denominators and create equivalent fractions in order to add rational expressions.

### Vocabulary

<sup>1</sup> mathematical phrases written with numbers and variables connected by operations

## Concept Development

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

- *Two rational expressions can only be added when they have **common denominators**.*

## Add Rational Expressions

$$\frac{x+3}{x-1} + \frac{x+5}{x-4}$$

The LCD is  
 $(x-1)(x-4)$

$$\frac{x-4}{x-4} \cdot \frac{x+3}{x-1} + \frac{x+5}{x-4} \cdot \frac{x-1}{x-1}$$

Create equivalent fractions

$$\frac{x^2 - x - 12}{(x-1)(x-4)} + \frac{x^2 + 4x - 5}{(x-1)(x-4)}$$

$$\frac{2x^2 + 3x - 17}{(x-1)(x-4)}$$

### CFU

Which of the following is ready to be added? How do you know?

A  $\frac{(x-1)}{(x-3)} + \frac{(x+4)}{(x+2)}$

B  $\frac{x+12}{(x-5)(x+4)} + \frac{4x-5}{(x-5)(x+4)}$

Why can the other not be added?

Which of the following correctly shows the addition of the expression below? How do you know?

$$\frac{4x}{x-3} + \frac{7x-1}{x-3}$$

A  $\frac{4x+7x-1}{x-3+x-3}$   B  $\frac{4x+7x-1}{x-3}$

In your own words, what is a rational expression?

"A rational expression is \_\_\_\_\_."

## Skill Development/Guided Practice

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

### Add rational expressions.

- 1 Factor the polynomials in the denominator completely, if possible.
- 2 Create equivalent fractions by finding a common denominator.
  - a Multiply the numerator and denominator of each fraction by the same rational expression.
- 3 Add the rational expressions.
  - a Combine like terms in the numerator, leaving the common denominator the same.
- 4 Simplify the sum, if possible.

### CFU

- 1 How did I/you factor the denominator?
- 2 How did I/you create equivalent fractions?
- 3 How did I/you add the rational expressions?
- 4 How did I/you simplify the sum?

$$1. \frac{x^2 + 3x - 7}{x + 3} + \frac{x + 10}{x + 3}$$

$$\frac{x^2 + 3x - 7 + x + 10}{x + 3} \quad \frac{(\cancel{x+3})(x+1)}{\cancel{x+3}}$$

$$\frac{x^2 + 4x + 3}{x + 3} \quad \boxed{x + 1}$$

$$2. \frac{x^2 + 3x - 8}{x + 6} + \frac{x - 4}{x + 6}$$

$$\frac{x^2 + 3x - 8 + x - 4}{x + 6} \quad \frac{(\cancel{x+6})(x-2)}{\cancel{x+6}}$$

$$\frac{x^2 + 4x - 12}{x + 6} \quad \boxed{x - 2}$$

$$3. \frac{7}{x + 3} + \frac{2}{x + 6}$$

$$\frac{x + 6}{x + 6} \cdot \frac{7}{x + 3} + \frac{2}{x + 6} \cdot \frac{x + 3}{x + 3}$$

$$\frac{7x + 42}{(x + 3)(x + 6)} + \frac{2x + 6}{(x + 3)(x + 6)}$$

$$\boxed{\frac{9x + 48}{(x + 3)(x + 6)}}$$

$$4. \frac{5}{x - 2} + \frac{3}{x + 5}$$

$$\frac{x + 5}{x + 5} \cdot \frac{5}{x - 2} + \frac{3}{x + 5} \cdot \frac{x - 2}{x - 2}$$

$$\frac{5x + 25}{(x + 5)(x - 2)} + \frac{3x - 6}{(x + 5)(x - 2)}$$

$$\boxed{\frac{8x + 19}{(x + 5)(x - 2)}}$$

## Skill Development/Guided Practice (continued)

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

### Add rational expressions.

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### CFU

- 1 How did I/you factor the denominator?
- 2 How did I/you create equivalent fractions?
- 3 How did I/you add the rational expressions?
- 4 How did I/you simplify the sum?

$$5. \quad \frac{x+6}{x+3} + \frac{x-5}{x-2}$$

$$\frac{x-2}{x-2} \cdot \frac{x+6}{x+3} + \frac{x-5}{x-2} \cdot \frac{x+3}{x+3}$$

$$\frac{x^2 + 4x - 12}{(x+3)(x-2)} + \frac{x^2 - 2x - 15}{(x+3)(x-2)}$$

$$\frac{2x^2 + 2x - 27}{(x+3)(x-2)}$$

$$7. \quad \frac{x+4}{x^2 - 2x - 8} + \frac{3}{x^2 - 6x + 8}$$

$$\frac{x-2}{x-2} \cdot \frac{x+4}{(x+2)(x-4)} + \frac{3}{(x-4)(x-2)} \cdot \frac{x+2}{x+2}$$

$$\frac{x^2 + 2x - 8}{(x+2)(x+4)(x-2)} + \frac{3x+2}{(x+2)(x+4)(x-2)}$$

$$\frac{x^2 + 5x - 6}{(x+2)(x-4)(x-2)}$$

$$6. \quad \frac{x+2}{x-1} + \frac{x-9}{x+5}$$

$$\frac{x+5}{x+5} \cdot \frac{x+2}{x-1} + \frac{x-9}{x+5} \cdot \frac{x-1}{x-1}$$

$$\frac{x^2 + 7x + 10}{(x+5)(x-1)} + \frac{x^2 - 10x + 9}{(x+5)(x-1)}$$

$$\frac{2x^2 - 3x + 19}{(x+3)(x-2)}$$

$$8. \quad \frac{x+2}{x^2 - x - 2} + \frac{7}{x^2 + x - 6}$$

$$\frac{x+3}{x+3} \cdot \frac{x+2}{(x-2)(x+1)} + \frac{7}{(x+3)(x-2)} \cdot \frac{x+1}{x+1}$$

$$\frac{x^2 + 5x + 6}{(x+3)(x-2)(x+1)} + \frac{7x+7}{(x+3)(x-2)(x+1)}$$

$$\frac{x^2 + 12x + 13}{(x+2)(x-4)(x-2)}$$

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

- 1 *Adding rational expressions will help you simplify more complex math problems.*

Simplify

$$\left( \frac{\frac{3}{x-1}}{\frac{5}{x+1} + \frac{1}{x^2-1}} \right) = \frac{3x+3}{5x-4}$$

- 2 *Adding rational expressions will help you do well on tests.*

Sample Test Question:

10.  $\frac{x^2 - 2x + 9}{x + 6} + \frac{x + 4}{x + 6} =$

A  $\frac{x^2 - x + 13}{2x + 12}$

B  $\frac{x^2 - 3x + 13}{x + 12}$

C  $\frac{x^2 - x + 13}{x + 6}$

D  $x^2 - x + 13$

CFU

Does anyone else have another reason why it is relevant to add rational expressions? (Pair-Share) Why is it relevant to add rational expressions? You may give one of my reasons or one of your own. Which reason is more relevant to you? Why?

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

### Skill Closure

#### Add rational expressions.

- 1 Factor the polynomials in the denominator completely, if possible.
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- 4 Simplify the sum, if possible.

1. 
$$\frac{x^2 + 7x + 4}{x + 1} + \frac{x + 3}{x + 1}$$

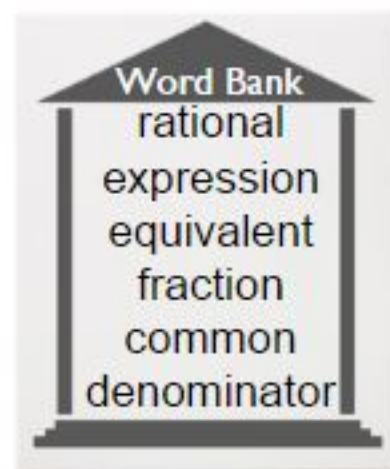
$$\frac{x^2 + 7x + 4 + x + 3}{x + 1} \quad \frac{\cancel{(x+1)}(x+7)}{\cancel{x+1}}$$
$$\frac{x^2 + 8x + 7}{x + 1} \quad \boxed{x + 7}$$

2. 
$$\frac{2}{x - 1} + \frac{3}{x + 7}$$

$$\frac{x + 7}{x + 7} \cdot \frac{2}{x - 1} + \frac{3}{x + 7} \cdot \frac{x - 1}{x - 1}$$
$$\frac{2x + 14}{(x + 7)(x - 1)} + \frac{3x - 3}{(x + 7)(x - 1)}$$
$$\boxed{\frac{5x + 11}{(x + 7)(x - 1)}}$$

### Summary Closure

What did you learn today about adding rational expressions? (Pair-Share)  
Use words from the word bank.



## Independent Practice

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

### Add rational expressions.

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$$1. \frac{x^2 + 7x - 10}{x - 1} + \frac{x + 1}{x - 1}$$

$$\frac{x^2 + 7x - 10 + x + 1}{x - 1} \quad \frac{\cancel{x-1}(x+9)}{\cancel{x-1}}$$

$$\frac{x^2 + 8x - 9}{x - 1} \quad \boxed{x + 9}$$

$$2. \frac{3}{x + 4} + \frac{3}{x - 7}$$

$$\frac{x - 7}{x - 7} \cdot \frac{3}{x + 4} + \frac{3}{x - 7} \cdot \frac{x + 4}{x + 4}$$

$$\frac{3x - 21}{(x - 7)(x + 4)} + \frac{3x + 12}{(x - 7)(x + 4)} \quad \boxed{\frac{6x - 9}{(x - 7)(x + 4)}}$$

$$3. \frac{x + 2}{x + 3} + \frac{x - 7}{x - 5}$$

$$\frac{x - 5}{x - 5} \cdot \frac{x + 2}{x + 3} + \frac{x - 7}{x - 5} \cdot \frac{x + 3}{x + 3}$$

$$\frac{x^2 - 3x - 10}{(x + 3)(x - 5)} + \frac{x^2 - 4x - 21}{(x + 3)(x - 5)} \quad \boxed{\frac{2x^2 - 7x - 31}{(x + 3)(x - 5)}}$$

$$4. \frac{x - 5}{x^2 + 3x + 2} + \frac{9}{x^2 + x - 2}$$

$$\frac{x - 1}{x - 1} \cdot \frac{x - 5}{(x + 2)(x + 1)} + \frac{9}{(x - 1)(x + 2)} \cdot \frac{x + 1}{x + 1}$$

$$\frac{x^2 - 6x + 5}{(x - 1)(x - 2)(x + 1)} + \frac{9x + 9}{(x - 1)(x - 2)(x + 1)} \quad \boxed{\frac{x^2 + 3x + 14}{(x + 2)(x - 4)(x - 2)}}$$



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$$1. \quad \frac{x^2 + 2x - 3}{x + 2} + \frac{3x - 7}{x + 2}$$

$$\frac{x^2 + 2x - 3 + 3x - 7}{x + 2}$$

$$\frac{x^2 + 5x - 10}{x + 2}$$

$$2. \quad \frac{7}{x + 3} + \frac{5}{x + 1}$$

$$\frac{x + 1}{x + 1} \cdot \frac{7}{x + 3} + \frac{5}{x + 1} \cdot \frac{x + 3}{x + 3}$$

$$\frac{7x + 7}{(x + 1)(x + 3)} + \frac{5x + 15}{(x + 1)(x + 3)}$$

$$\frac{12x + 22}{(x - 7)(x + 4)}$$

$$3. \quad \frac{x - 1}{x + 3} + \frac{x + 4}{x + 1}$$

$$\frac{x + 1}{x + 1} \cdot \frac{x - 1}{x + 3} + \frac{x + 4}{x + 1} \cdot \frac{x + 3}{x + 3}$$

$$\frac{x^2 - 1}{(x + 3)(x + 1)} + \frac{x^2 + 7x + 12}{(x + 3)(x + 1)}$$

$$\frac{2x^2 + 7x + 11}{(x + 3)(x + 1)}$$

$$4. \quad \frac{x + 1}{x^2 + 3x - 10} + \frac{4}{x^2 - 9x + 14}$$

$$\frac{x - 7}{x - 7} \cdot \frac{x + 1}{(x + 5)(x - 2)} + \frac{4}{(x - 7)(x - 2)} \cdot \frac{x + 5}{x + 5}$$

$$\frac{x^2 - 6x - 7}{(x - 1)(x - 2)(x + 5)} + \frac{4x + 20}{(x - 1)(x - 2)(x + 5)}$$

$$\frac{x^2 - 2x + 13}{(x - 1)(x - 2)(x + 5)}$$

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- 4 Simplify the sum, if possible.

$$1. \quad \frac{x+1}{x+3} + \frac{x-2}{x+4}$$

$$\frac{x+4}{x+4} \cdot \frac{x+1}{x+3} + \frac{x-2}{x+4} \cdot \frac{x+3}{x+3}$$

$$\frac{x^2+5x+4}{(x+3)(x+4)} + \frac{x^2+x-6}{(x+3)(x+4)}$$

$$\frac{2x^2+6x-2}{(x+3)(x+4)}$$

$$3. \quad \frac{5}{x+1} + \frac{2}{5x+3}$$

$$\frac{5x+3}{5x+3} \cdot \frac{5}{x+1} + \frac{2}{5x+3} \cdot \frac{x+1}{x+1}$$

$$\frac{25x+15}{(x+1)(5x+3)} + \frac{2x+2}{(x+1)(5x+3)}$$

$$\frac{27x+17}{(x+1)(5x+3)}$$

$$2. \quad \frac{x+2}{x^2-1} + \frac{1}{x^2+3x+2}$$

$$\frac{x^2+5x+5}{(x+2)(x+1)(x-1)}$$

$$\frac{x+2}{x+2} \cdot \frac{x+2}{(x+1)(x-1)} + \frac{1}{(x-1)(x+2)} \cdot \frac{x+1}{x+1}$$

$$\frac{x^2+4x+4}{(x+2)(x+1)(x-1)} + \frac{x+1}{(x+2)(x+1)(x-1)}$$

$$4. \quad \frac{x^2-4x-7}{x+1} + \frac{5x+7}{x+1}$$

$$\frac{x^2-4x-7+5x+7}{x+1}$$

$$\frac{x^2+x}{x+1}$$

$$\frac{x(\cancel{x+1})}{\cancel{x+1}}$$

$$x$$

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- 4 Simplify the sum, if possible.

$$1. \quad \frac{x^2 - 8x + 6}{x - 1} + \frac{x + 1}{x - 1}$$

$$\frac{x^2 - 8x + 6 + x + 1}{x - 1}$$

$$\frac{x^2 - 7x + 7}{x - 1}$$

$$2. \quad \frac{x + 7}{x - 1} + \frac{x - 3}{x + 2}$$

$$\frac{x + 2}{x + 2} \cdot \frac{x + 7}{x - 1} + \frac{x - 3}{x + 2} \cdot \frac{x - 1}{x - 1}$$

$$\frac{x^2 + 9x + 14}{(x + 2)(x - 1)} + \frac{x^2 - 4x + 3}{(x + 2)(x - 1)}$$

$$\frac{2x^2 + 5x + 17}{(x + 2)(x - 1)}$$

$$3. \quad \frac{x - 5}{x^2 + 3x + 2} + \frac{9}{x^2 + x - 2}$$

$$\frac{x^2 + 3x + 14}{(x + 2)(x + 1)(x - 1)}$$

$$\frac{x - 1}{x - 1} \cdot \frac{x - 5}{(x + 1)(x + 2)} + \frac{9}{(x - 1)(x + 2)} \cdot \frac{x + 1}{x + 1}$$

$$\frac{x^2 - 6x + 5}{(x + 2)(x + 1)(x - 1)} + \frac{9x + 9}{(x + 2)(x + 1)(x - 1)}$$

$$4. \quad \frac{3}{x + 4} + \frac{-3}{x - 2}$$

$$\frac{x - 2}{x - 2} \cdot \frac{3}{x + 4} + \frac{-3}{x - 2} \cdot \frac{x + 4}{x + 4}$$

$$\frac{3x - 6}{(x - 2)(x + 4)} + \frac{-3x - 12}{(x - 2)(x + 4)}$$

$$\frac{-18}{(x - 2)(x + 4)}$$