Review for Exam-2: Proofs with Lines and Angles

1 **Find the equation of the line described**

Perpendicular to $y = 2x + 4$; passing through the point (5, 6).

Perpendicular lines have opposite reciprocal slope, so $m = \frac{-1}{2}$.

Use point-slope form.

$$(y - y_1) = m(x - x_1)$$

$$(y - 6) = \frac{-1}{2}(x - 5)$$ Substitute for m, x_1, and y_1.

$$y - 6 = \frac{-x}{2} + \frac{5}{2}$$ Distribute.

*Note $-5\left(\frac{-1}{2}\right) = \frac{5}{2}$

$+6 \quad +6$

$$y = \frac{-x}{2} + \frac{17}{2}$$ Solve for y.

*Note $\frac{5}{2} + 6 = 6\frac{5}{2} = \frac{6(2)+5}{2} = \frac{17}{2}$ Write as improper fractions

2 Find the equation of the line.

The line is perpendicular to $y = \frac{1}{6}x + 8$ and passes through the point (2, 7).

The slope of the perpendicular line is -6.

Let $(x_1, y_1) = (2, 7)$ and $m = -6$.

Point-slope form

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -6(x - 2) \quad \text{Substitute } m, x_1, \text{ and } y_1.$$

$$y - 7 = -6x + 12 \quad \text{Simplify.}$$
$$+7 \qquad\quad +7$$

$$y = -6x + 19 \quad \text{Solve for } y.$$

The equation of the line is $y = -6x + 19$.

3 Find the equation of the line.

The line is parallel to $y = -\frac{1}{7}x + 6$ and passes through the point $(-7, -2)$.

The slope of the parallel line is SAME.

Let $(x_1, y_1) = (-7, -2)$ and $m = -\frac{1}{7}$.

Point-slope form

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{1}{7}(x - (-7))$$ Substitute m, x_1, and y_1.

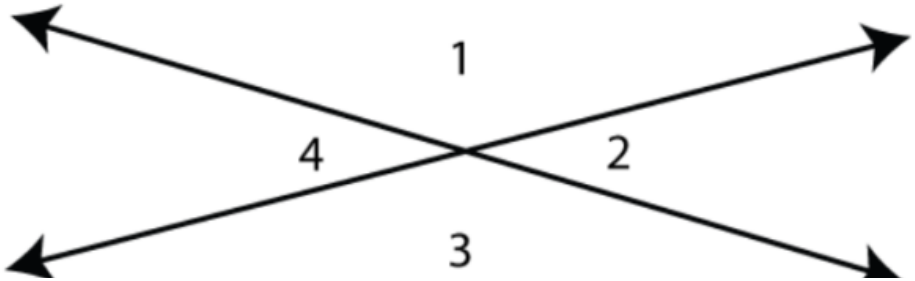
$$y + 2 = -\frac{1}{7}x - 1$$ Simplify.

$$y = -\frac{1}{7}x - 3$$ Solve for y.

The equation of the line is $y = -\frac{1}{7}x - 3$.

4 Part 1 out of 3

Complete a proof of the Vertical Angles Theorem in paragraph proof form.



Given: ∠2 and ∠4 are vertical angles.
Prove: ∠2 ≅ ∠4
In the diagram of intersecting lines, ∠2 and ∠4 ▼ are vertical angles.
Also, ∠2 and ∠3 are a linear pair and ∠3 and ∠4 ▼ are a linear pair.

Part 2 ✓

By the Linear Pair Theorem, ∠2 and ∠3 are supplementary and ∠3 and ∠4 are supplementary ▼.
Then m∠2 + m∠3 = 180 ° and m∠3 + m∠4 = 180 ° by the definition of supplementary angles.

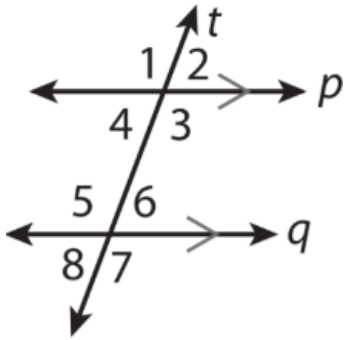
Part 3 ✓

By the Transitive Property of Equality, m∠2 + m∠3 = ▼ m∠3 + m∠4.
Using the Subtraction Property of Equality, m∠2 = m∠4.
So, ∠2 ≅ ∠4 by the definition of congruence ▼.

5 Complete the proof in two-column form for the Corresponding Angles Theorem.

Given: $p \parallel q$

Prove: $m\angle 7 = m\angle 3$



Part 1 ✓

Statements:	Reasons:
1. $p \parallel q$	1. Given

Part 2 ✓

Statements:	Reasons:
2. $m\angle 5$ $= m\angle 3$	2. Alternate Interior Angles Theorem

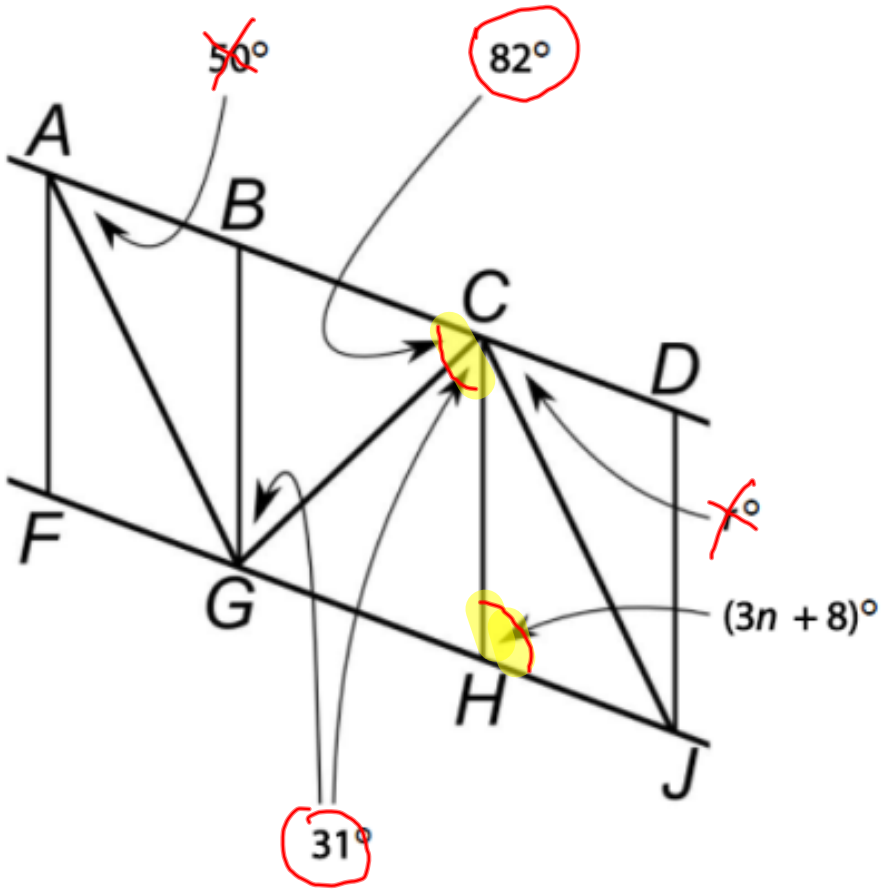
Part 3 ✓

Statements:	Reasons:
3. $m\angle 7 =$ $m\angle 5$	3. Vertical Angles Theorem

Part 4 ✓

Statements:	Reasons:
4. $m\angle 7 = m\angle 3$	4. Substitution Property of Equality

6 In the diagram of the staircase railing below, $\overline{AG} \parallel \overline{CJ}$ and $\overline{AD} \parallel \overline{FJ}$.

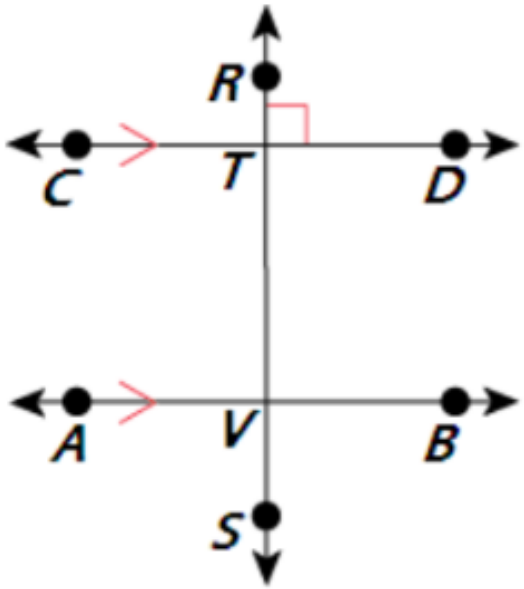


$$\begin{aligned} 3n + 8 &= 82 + 31 \\ 3n + 8 &= 113 \\ -8 & \quad -8 \\ \hline 3n &= 105 \\ \frac{3n}{3} &= \frac{105}{3} \\ n &= 35 \end{aligned}$$

Find the value of n.

$n =$ 35, by the Alternate Interior Angles Theorem ▼.

7 Prove the theorem: In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.



Given the facts $\overline{RS} \perp \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$, drag and drop each reason next to the appropriate statement in order to prove the statement $\overline{RS} \perp \overline{AB}$.

Statements	Reasons
$\overline{AB} \parallel \overline{CD}$	Given
$m\angle RTD = m\angle RVB$	Corresponding Angles Theorem
$\overline{RS} \perp \overline{CD}$	Given
$m\angle RTD = 90°$	Definition of perpendicular lines
$m\angle RVB = 90°$	Substitution Property of Equality
$\overline{RS} \perp \overline{AB}$	Definition of perpendicular lines

8 The measures of two vertical angles are given by the expressions $(x + 5)°$ and $(2x - 7)°$.

Find the value of x. What is the measure of each angle?

Vertical angles are congruent.

Set vertical angles equal.

$$x + 5 = 2x - 7$$

$+7 \qquad +7$ Add 7 to both sides.

$$x + 12 = 2x$$

Solve for x.

$$12 = x$$

Find the measure of each angle.

$$(x + 5)° = (12 + 5)° = 17°$$

9 You can represent the measures of an angle and its complement as $x°$ and $(90 - x)°$. Similarly, you can represent the measures of an angle and its supplement as $x°$ and $(180 - x)°$. Use these expressions to find the measures of the angles described.

The measure of an angle is equal to three times the measure of its complement.

$$x = 3(90 - x)$$

$$x = 270 - 3x$$

$+3x \qquad +3x$ Add $3x$ to both sides.

$$4x = 270$$

$$x = 67.5$$ Solve for x.

The measure of the angle is 67.5° and the measure of its complement is 22.5°.

$90 - 67.5 =$

10 You can represent the measures of an angle and its complement as x° and (90 x)°. Similarly, you can represent the measures of an angle and its supplement as x° and (180 x)°. Use these expressions to find the measures of the angles

The measure of an angle is equal to five times the measure of its supplement.

$$x = 5(180 - x)$$

$$x = 900 - 5x$$

$+5x \qquad +5x$ Add $5x$ to both sides.

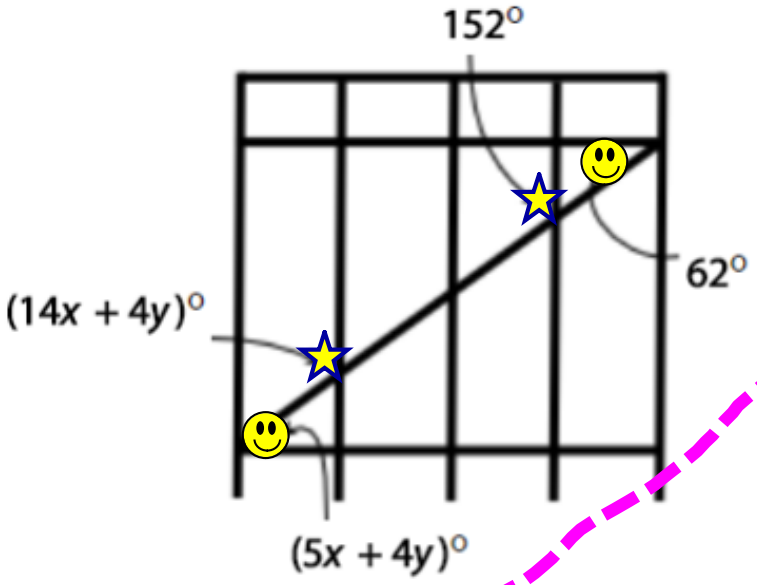
$$6x = 900$$ Solve for x.

$$x = 150$$

The measure of the angle is 150 ° and the measure of its supplement is 30 °.

180 − 150 = 30

11 In the diagram of a gate, the horizontal bars are parallel and the vertical bars are parallel. Find x and y. Complete the explanation indicating which postulates and/or theorems were used to find the values.



System of Equations

$14x + 4y° = 152°$

$-5x + 4y° = 62°$

$\frac{9x}{9} = \frac{90}{9}$

$x = 10$

Find the y, $5x + 4y° = 62°$

$50 + 4y = 62$

$-50 \quad -50$

$\frac{4y}{4} = \frac{12}{4}$

$y = 3$

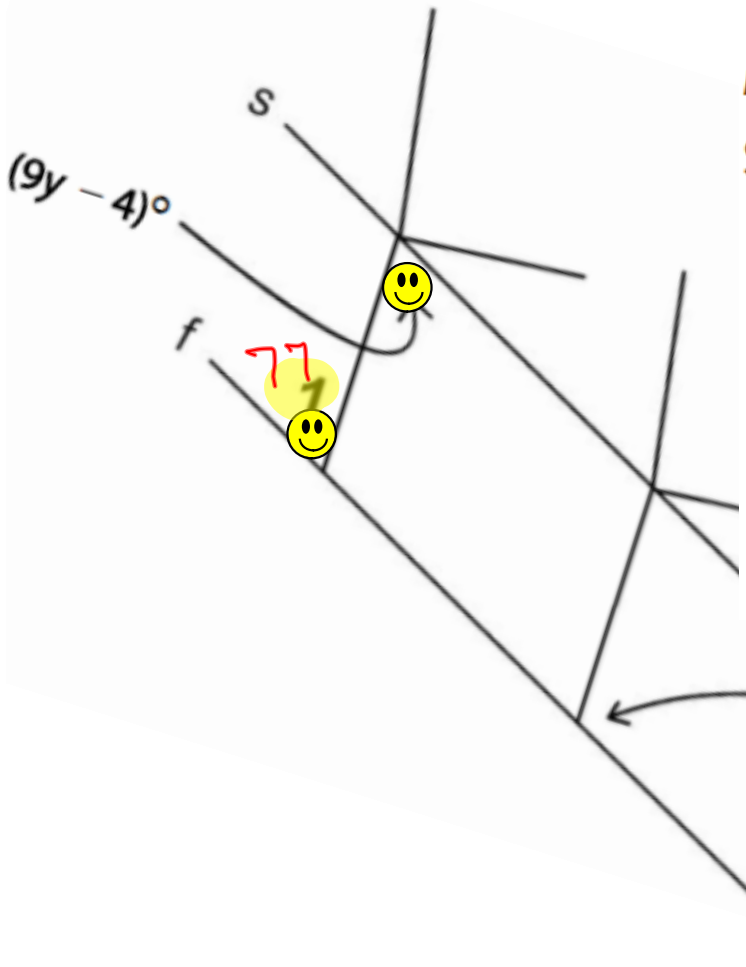
$x =$ 10

$y =$ 3

$(14x + 4y)° =$ 152 ° by the Corresponding Angles Theorem ▼ an

$(5x + 4y)° =$ 62 ° by the Alternate Interior Angles Theorem ▼ .

12 In the diagram of movie theater seats, the incline of the floor, f, is parallel to the seats, s.



If $m\angle 1 = 77°$, what is y?

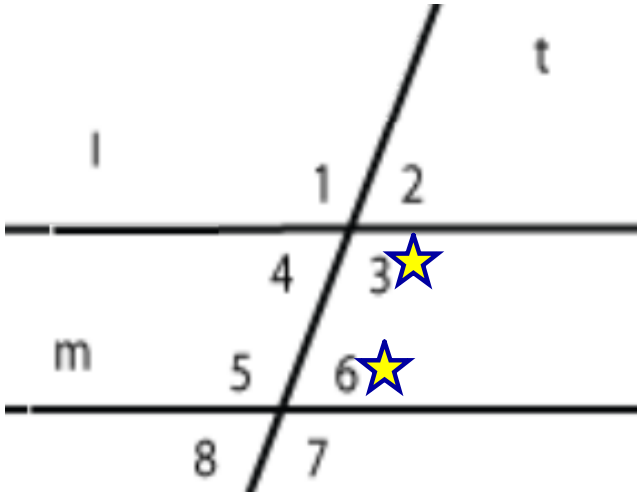
$m\angle 1 = (9y - 4)°$, by the Alternate Int. Angles The

Substitute $77°$ for $m\angle 1$ and solve for y:

$$\begin{aligned} m\angle 1 &= (9y - 4)° \\ 77 &= 9y - 4 \\ +4 & \quad +4 \\ \hline 81 &= 9y \\ 9 &= y \end{aligned}$$

So, $y = 9$, by the Alternate Int. Angles Theorem.

13 Suppose $m\angle 6 = (2x + 5)°$ and $m\angle 3 = (2x + 95)°$, where $x = 20$.
Are the lines parallel? Complete the explanation.



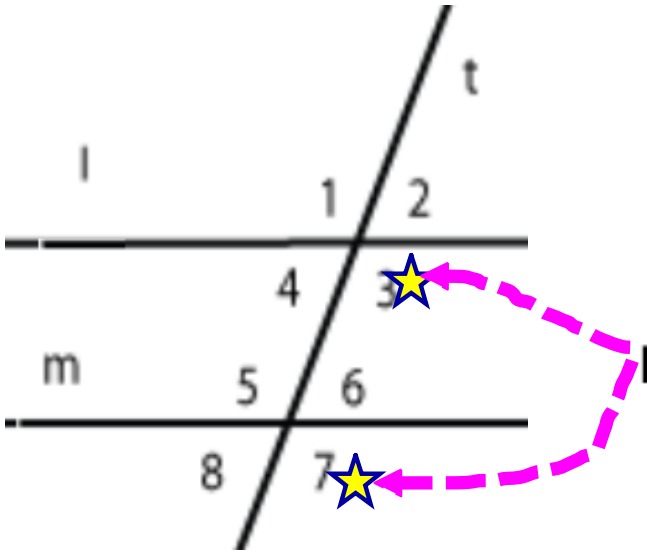
$m\angle 6 = (2(20) + 5)° = 45°$

$m\angle 3 = (2(20) + 95)° = 135°$

$m\angle 6 =$ 45 ° and $m\angle 3 =$ 135 ° so, $m\angle 6 + m\angle 3 =$ 180 °.

Yes ▾, the lines are ▾ parallel by the Converse of the Same–Side Interior Angles Postulate ▾

14 Suppose $m\angle 3 = (3x + 7)°$ and $m\angle 7 = (85 - 2x)°$, where $x = 15$.
Are the lines parallel? Complete the explanation.



$m\angle 3 = (3(15) + 7)° = 52°$

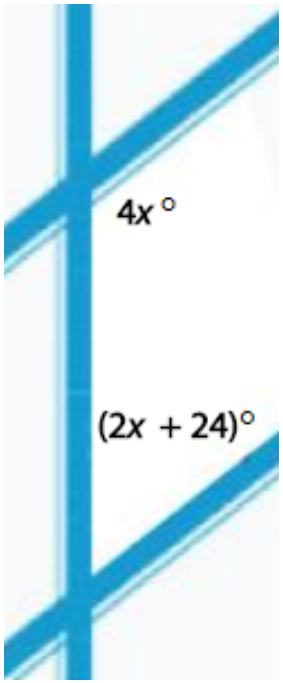
$m\angle 7 = (85 - 2(15))° = 55°$

pair of corresponding angles

$m\angle 3 =$ 52 ° and $m\angle 7 =$ 55 ° so, $m\angle 3$ ≠ ▾ $m\angle 7$.

No ▾, the lines are not ▾ parallel because a pair of corresponding angles are not ▾ congruent

15 An overpass intersects two lanes of a highway. What must the value of x be to ensure the two lanes are parallel?



The two angles between the lanes of a highway are same side interior angles. Therefore, they must be supplementary to make the lanes parallel.

$$\begin{aligned} 2x + 24 + 4x &= 180 \\ 6x + 24 &= 180 \\ -24 \quad &\quad -24 \\ \hline 6x &= 156 \\ x &= 26 \end{aligned}$$

When $x =$ 26, the same-side interior angles are supplementary and the lanes are parallel.