



Simplify Rational Expressions by **FACTORING**

Learning Objective

We will simplify¹ rational expressions by factoring.

CFU

What are we going to learn?

What does *simplify* mean?
Simplify means _____.

Activate Prior Knowledge

Factoring a polynomial is writing a polynomial as a product of two smaller polynomial factors.

Factor the polynomials by finding the common factors.

1. $5x^2 + 35x$

$$\boxed{5} \cdot \boxed{x} \cdot x + 7 \cdot \boxed{5} \cdot \boxed{x}$$

$$5x(x + 7)$$

2. $2x^2 - 6x$

$$\boxed{2} \cdot \boxed{x} \cdot x - 3 \cdot \boxed{2} \cdot \boxed{x}$$

$$2x(x - 3)$$

Factor the trinomials.

3. $x^2 + 3x - 28$ $(x - 4)(x + 7)$

-28

$$28 - 1 = 27$$

$$14 - 2 = 12$$

$$7 - 4 = 3$$

4. $x^2 + 3x - 18$ $(x - 3)(x + 6)$

-18

$$18 - 1 = 17$$

$$9 - 2 = 7$$

$$6 - 3 = 3$$

Make Connection

Students, you already know how to factor polynomials. Now, we will use your skill in factoring polynomials to simplify rational expressions.

Vocabulary

¹ reduce to lowest terms

Concept Development

A **polynomial** is an expression² of one or more terms.

$$7x^5$$

monomial

$$4x^2 + 3x$$

binomial

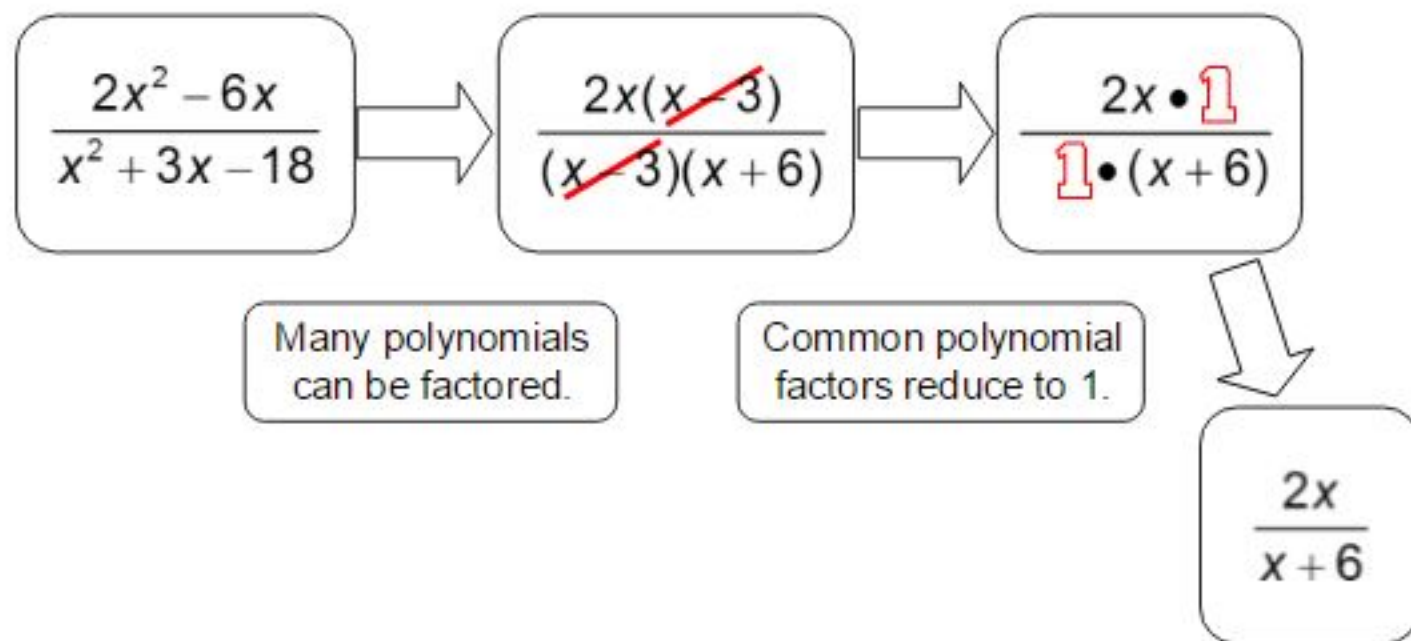
$$x^3 + x^2 - x$$

trinomial

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

- Rational expressions can be simplified by factoring the polynomials and reducing the common polynomial factors.

Simplifying Rational Expressions



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On your whiteboards, write a fraction with the polynomial $15x^2 - 4x + 3$ in the numerator and $13x - 7$ in the denominator.

Which rational expression can be reduced? How do you know?

A $\frac{(x-3)(x+4)}{(x-3)(x+2)}$

B $\frac{(x-3)(x+5)}{(x+3)(x+2)}$

In your own words, what is a polynomial?

"A polynomial is _____."

In your own words, what is a rational expression?

"A rational expression is _____."

Vocabulary

² mathematical phrase written with numbers and variables connected by operations

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

Simplify the rational expressions by factoring.

- Factor the polynomial in the numerator completely.
- Factor the polynomial in the denominator completely.
- Simplify the fraction by reducing common polynomial factors in the numerator and denominator.

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- How did I/you know how to factor the numerator?
- How did I/you know how to factor the denominator?
- How did I/you reduce the common polynomial factors?

$$1. \quad \frac{3x^2 - 21x}{2x - 14} = \frac{\cancel{3x(x-7)}}{\cancel{2(x-7)}} = \frac{3x}{2}$$

$$\begin{array}{l} 3x^2 - 21x \\ \boxed{3} \cdot \boxed{x} \cdot x - 7 \cdot \boxed{3} \cdot \boxed{x} \\ 3x(x-7) \end{array}$$

$$\begin{array}{l} 2x - 14 \\ \boxed{2} \cdot x - 7 \cdot \boxed{2} \\ 2(x-7) \end{array}$$

$$2. \quad \frac{5x^2 + 15x}{7x + 21} = \frac{\cancel{5x(x+3)}}{\cancel{7(x+3)}} = \frac{5x}{7}$$

$$\begin{array}{l} 5x^2 + 15x \\ \boxed{5} \cdot \boxed{x} \cdot x + 3 \cdot \boxed{5} \cdot \boxed{x} \\ 5x(x+3) \end{array}$$

$$\begin{array}{l} 7x + 21 \\ \boxed{7} \cdot x + 3 \cdot \boxed{7} \\ 7(x+3) \end{array}$$

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

Simplify the rational expressions by factoring.

- Factor the polynomial in the numerator completely.
- Factor the polynomial in the denominator completely.
- Simplify the fraction by reducing common polynomial factors in the numerator and denominator.

CFU

- How did I/you know how to factor the numerator?
- How did I/you know how to factor the denominator?
- How did I/you reduce the common polynomial factors?

$$3. \quad \frac{x^2 + 11x + 28}{x^2 + x - 12} = \frac{\cancel{(x+4)}(x+7)}{\cancel{(x+4)}(x-3)} = \frac{x+7}{x-3}$$

$x^2 + 11x + 28$	$x^2 + x - 12$
<u>28</u>	<u>-12</u>
$28 + 1 = 4$	$12 - 1 = 11$
$14 + 2 = 16$	$6 - 2 = 4$
<u>$7 + 4 = 11$</u>	<u>$4 - 3 = 1$</u>
$(x+4)(x+7)$	$(x+4)(x-3)$

$$4. \quad \frac{x^2 - 2x - 15}{x^2 - 11x + 30} = \frac{\cancel{(x-5)}(x+3)}{\cancel{(x-5)}(x-6)} = \frac{x+3}{x-6}$$

$x^2 - 2x - 15$	$x^2 - 11x + 30$
<u>-15</u>	<u>30</u>
$15 - 1 = 14$	$30 + 1 = 31$
<u>$5 - 3 = 2$</u>	$15 + 2 = 17$
$(x-5)(x+3)$	$10 + 3 = 13$
	<u>$6 + 5 = 11$</u>
	$(x-6)(x-5)$

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

Simplify the rational expressions by factoring.

- Factor the polynomial in the numerator completely.
- Factor the polynomial in the denominator completely.
- Simplify the fraction by reducing common polynomial factors in the numerator and denominator.

CFU

- How did I/you know how to factor the numerator?
- How did I/you know how to factor the denominator?
- How did I/you reduce the common polynomial factors?

$$5. \quad \frac{7x^2 + 21x}{x^2 + 10x + 21} = \frac{\cancel{7x(x+3)}}{\cancel{(x+3)}(x+7)} = \frac{7x}{x+7}$$

$$\begin{array}{r} 7x^2 + 21x \\ \boxed{7} \cdot \boxed{x} \cdot x + 3 \cdot \boxed{7} \cdot \boxed{x} \\ 7x(x+3) \end{array} \qquad \begin{array}{r} x^2 + 10x + 21 \\ \underline{21} \\ 21 + 1 = 22 \\ \boxed{7 + 3 = 10} \\ (x+3)(x+7) \end{array}$$

$$6. \quad \frac{5x^2 - 25x}{x^2 - 10x + 25} = \frac{\cancel{5x(x-5)}}{\cancel{(x-5)}(x-5)} = \frac{5x}{x-5}$$

$$\begin{array}{r} 5x^2 - 25x \\ \boxed{5} \cdot \boxed{x} \cdot x - 5 \cdot \boxed{5} \cdot \boxed{x} \\ 5x(x-5) \end{array} \qquad \begin{array}{r} x^2 - 10x + 25 \\ \underline{25} \\ 25 + 1 = 26 \\ \boxed{5 + 5 = 10} \\ (x-5)(x-5) \end{array}$$

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

Simplify the rational expressions by factoring.

- Factor the polynomial in the numerator completely.
- Factor the polynomial in the denominator completely.
- Simplify the fraction by reducing common polynomial factors in the numerator and denominator.

CFU

- How did I/you know how to factor the numerator?
- How did I/you know how to factor the denominator?
- How did I/you reduce the common polynomial factors?

$$7. \quad \frac{x^2 + 5x - 6}{x^2 - 4x - 12} = \frac{(x+6)(x-1)}{(x-6)(x+2)}$$

$$x^2 + 5x - 6$$

$$\begin{array}{r} -6 \\ \hline \end{array}$$

$$6 - 1 = 5$$

$$3 - 2 = 1$$

$$(x+6)(x-1)$$

$$x^2 - 4x - 12$$

$$\begin{array}{r} -12 \\ \hline \end{array}$$

$$12 - 1 = 11$$

$$6 - 2 = 4$$

$$4 - 3 = 1$$

$$(x-6)(x+2)$$

$$8. \quad \frac{x^2 - 5x - 14}{x^2 + 2x - 8} = \frac{(x-7)(x+2)}{(x-2)(x+4)}$$

$$x^2 - 5x - 14$$

$$\begin{array}{r} -14 \\ \hline \end{array}$$

$$14 - 1 = 13$$

$$7 - 2 = 5$$

$$(x-7)(x+2)$$

$$x^2 + 2x - 8$$

$$\begin{array}{r} -8 \\ \hline \end{array}$$

$$8 - 1 = 7$$

$$4 - 2 = 2$$

$$(x-2)(x+4)$$

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

- 1 *Simplifying rational expressions will help you multiply and divide rational expressions.*

$$\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+4)} \cdot \frac{\cancel{(x-3)}(x+5)}{\cancel{(x-3)}(x+2)} = \frac{(x+5)}{(x+4)}$$

- 2 *Simplifying rational expressions will help you do well on tests.*

Sample Test Question:

32. What is $\frac{x^2 - 4x + 4}{x^2 - 3x + 2}$ reduced to lowest terms?

- A $\frac{x-2}{x-1}$
- B $\frac{x-2}{x+1}$
- C $\frac{x+2}{x-1}$
- D $\frac{x+2}{x+1}$

CFU

Does anyone else have another reason why it is relevant to simplify rational expressions? (Pair-Share) Why is it relevant to simplify rational expressions? You may give one of my reasons or one of your own. Which reason is more relevant to you? Why?

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

Skill Closure

Simplify the rational expressions by factoring.

- Factor the polynomial in the numerator completely.
- Factor the polynomial in the denominator completely.
- Simplify the fraction by reducing common polynomial factors in the numerator and denominator.

Word Bank

polynomial
factoring
simplify
rational
expression

$$1. \quad \frac{7x^2 + 14x}{2x + 4} = \frac{\cancel{7x(x+2)}}{\cancel{2(x+2)}} = \frac{7x}{2}$$

$$\begin{array}{c} 7x^2 + 14x \\ \boxed{7} \cdot \boxed{x} \cdot x + 2 \cdot \boxed{7} \cdot \boxed{x} \\ 7x(x+2) \end{array}$$

$$\begin{array}{c} 2x + 4 \\ \boxed{2} \cdot x + 2 \cdot \boxed{2} \\ 2(x+2) \end{array}$$

$$2. \quad \frac{x^2 + 13x + 36}{x^2 + 7x - 18} = \frac{\cancel{(x+9)}(x+4)}{\cancel{(x-2)}(x+9)} = \frac{x+4}{x-2}$$

$$\begin{array}{r} x^2 + 13x + 36 \\ \quad \quad \quad 36 \\ \hline 36 + 1 = 37 \\ 18 + 2 = 20 \\ 12 + 3 = 15 \\ \underline{9 + 4 = 13} \\ 6 + 6 = 12 \\ (x+9)(x+4) \end{array}$$

$$\begin{array}{r} x^2 + 7x - 18 \\ \quad \quad \quad -18 \\ \hline 18 - 1 = 17 \\ \underline{9 - 2 = 7} \\ 6 - 3 = 3 \\ (x-2)(x+9) \end{array}$$

Summary Closure

What did you learn today about simplifying rational expressions by factoring? (Pair-Share)

Use words from the word bank.

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

Simplify the rational expressions by factoring.

- Factor the polynomial in the numerator completely.
- Factor the polynomial in the denominator completely.
- Simplify the fraction by reducing common polynomial factors in the numerator and denominator.

$$1. \frac{3x^2 + 15x}{2x + 10} = \frac{\cancel{3x(x+5)}}{\cancel{2(x+5)}} = \frac{3x}{2}$$

$$\begin{array}{c} 3x^2 + 15x \\ \boxed{3} \cdot \boxed{x} \cdot x + 5 \cdot \boxed{3} \cdot \boxed{x} \\ 3x(x+5) \end{array}$$

$$\begin{array}{c} 2x + 10 \\ \boxed{2} \cdot x + 5 \cdot \boxed{2} \cdot x \\ 2(x+5) \end{array}$$

$$2. \frac{x^2 + 6x - 7}{x^2 + 12x + 35} = \frac{\cancel{(x-7)}(x-1)}{\cancel{(x-7)}(x+5)} = \frac{x-1}{x+5}$$

$$\begin{array}{c} x^2 + 6x - 7 \\ -7 \\ \hline \boxed{7-1=6} \\ (x+7)(x-1) \end{array}$$

$$\begin{array}{c} x^2 + 12x + 35 \\ 35 \\ \hline 35 + 1 = 36 \\ \boxed{7+5=12} \\ (x+7)(x+5) \end{array}$$

Independent Practice (continued)

A **rational expression** is a fraction whose numerator and denominator contain polynomials.

Simplify the rational expressions by factoring.

- 1 Factor the polynomial in the numerator completely.
- 2 Factor the polynomial in the denominator completely.
- 3 Simplify the fraction by reducing common polynomial factors in the numerator and denominator.

$$3. \frac{7x^2 + 14x}{x^2 + 3x + 2} = \frac{\cancel{7x(x+2)}}{\cancel{(x+2)}(x+1)} = \frac{7x}{x+1}$$

$$\begin{array}{l} 7x^2 + 14x \\ \boxed{7} \cdot \boxed{x} \cdot x + 2 \cdot \boxed{7} \cdot \boxed{x} \\ 7x(x+2) \end{array} \quad \begin{array}{l} x^2 + 3x + 2 \\ 2 \\ \frac{2}{2+1=3} \\ (x+2)(x+1) \end{array}$$

$$4. \frac{x^2 + 6x - 7}{x^2 - 2x - 3} = \frac{(x+7)(x-1)}{(x+1)(x-3)}$$

$$\begin{array}{l} x^2 + 6x - 7 \\ -7 \\ \frac{-7}{7-1=6} \\ (x+7)(x-1) \end{array} \quad \begin{array}{l} x^2 - 2x - 3 \\ -3 \\ \frac{-3}{3-1=2} \\ (x+1)(x-3) \end{array}$$

$$1. \frac{9x^2 - 18x}{4x - 8} = \frac{9x(x-2)}{4(x-2)} = \frac{9x}{4}$$

$$\begin{array}{cc} 9x^2 - 18x & 4x - 8 \\ \boxed{3} \cdot \boxed{3} \cdot \boxed{x} \cdot x - 2 \cdot \boxed{3} \cdot \boxed{3} \cdot \boxed{x} & \boxed{2} \cdot \boxed{2} \cdot x - \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot x \\ 9x(x-2) & 4(x-2) \end{array}$$

$$2. \frac{x^2 - 4x + 3}{x^2 + 3x - 4} = \frac{(x-3)(x-1)}{(x+4)(x-1)} = \frac{x-3}{x+4}$$

$$\begin{array}{cc} x^2 - 4x + 3 & x^2 + 3x - 4 \\ 3 & -4 \\ \hline \boxed{3 + 1 = 4} & \boxed{4 - 1 = 3} \\ 2 - 2 = 0 & \\ (x-3)(x-1) & (x+4)(x-1) \end{array}$$

$$3. \frac{x^2 + 3x}{x^2 + x - 6} = \frac{x(x+3)}{(x-2)(x+3)} = \frac{x}{x-2}$$

$$\begin{array}{cc} x^2 + 3x & x^2 + x - 6 \\ \boxed{x} \cdot x + 3 \cdot \boxed{x} & \\ x(x+3) & \\ \hline 6 - 1 = 5 & \\ \boxed{3 - 2 = 1} & \\ (x-2)(x+3) & \end{array}$$

$$4. \frac{x^2 + 6x - 7}{x^2 + 12x - 45} = \frac{(x+7)(x-1)}{(x+3)(x-15)}$$

$$\begin{array}{cc} x^2 + 6x - 7 & x^2 - 12x - 45 \\ -7 & -45 \\ \hline \boxed{7 - 1 = 6} & \boxed{15 - 3 = 12} \\ 9 - 5 = 4 & \\ (x+7)(x-1) & (x+3)(x-15) \end{array}$$

$$1. \frac{x^2 + 6x - 16}{x^2 + 3x - 10} = \frac{(x+8)\cancel{(x-2)}}{(x+5)\cancel{(x-2)}} = \frac{x+8}{x+5}$$

$$x^2 + 6x - 16$$

$$\underline{-16}$$

$$16 - 1 = 15$$

$$8 - 2 = 6$$

$$4 - 4 = 0$$

$$(x+8)(x-2)$$

$$x^2 + 3x - 10$$

$$\underline{-10}$$

$$10 - 1 = 9$$

$$5 - 2 = 3$$

$$(x+5)(x-2)$$

$$2. \frac{x^2 + 14x + 45}{x^2 + 14x + 45} = \frac{(x+5)(x+9)}{(x-5)(x-9)}$$

$$x^2 + 14x + 45$$

$$\underline{45}$$

$$45 + 1 = 46$$

$$15 + 3 = 18$$

$$9 + 5 = 14$$

$$(x+5)(x+9)$$

$$x^2 - 14x + 45$$

$$\underline{45}$$

$$45 + 1 = 46$$

$$15 + 3 = 18$$

$$9 + 5 = 14$$

$$(x-9)(x-5)$$

$$3. \frac{5x^2 + 35x}{x^2 + 2x - 35} = \frac{5x\cancel{(x+7)}}{(x-5)\cancel{(x+7)}} = \frac{5x}{x-5}$$

$$5x^2 + 35x$$

$$5 \cdot x \cdot x + 7 \cdot 5 \cdot x$$

$$5x(x+7)$$

$$x^2 + 2x - 35$$

$$\underline{-35}$$

$$35 - 1 = 34$$

$$7 - 5 = 2$$

$$(x-5)(x+7)$$

$$4. \frac{7x^2 + 21x}{3x + 9} = \frac{7x\cancel{(x+3)}}{3\cancel{(x+3)}} = \frac{7x}{3}$$

$$7x^2 + 21x$$

$$7 \cdot x \cdot x + 3 \cdot 7 \cdot x$$

$$7x(x+3)$$

$$3x + 9$$

$$3 \cdot x + 3 \cdot 3$$

$$3(x+3)$$

$$1. \quad \frac{x^2 + 5x - 36}{x^2 - 3x + 28} = \frac{(x-4)(x+9)}{(x-7)(x+4)}$$

$$x^2 + 5x - 36$$

$$x^2 - 3x - 28$$

$$\underline{-36}$$

$$\underline{-28}$$

$$36 - 1 = 35$$

$$28 - 1 = 27$$

$$18 - 2 = 16$$

$$14 - 2 = 12$$

$$12 - 3 = 9$$

$$\underline{7 - 4 = 3}$$

$$\underline{9 - 4 = 5}$$

$$6 - 6 = 0$$

$$(x-7)(x+4)$$

$$(x-4)(x+9)$$

$$2. \quad \frac{x^2 + 4x + 3}{x^2 + x} = \frac{\cancel{(x+1)}(x+3)}{x\cancel{(x+1)}} = \frac{x+3}{x}$$

$$x^2 + 4x + 3$$

$$x^2 + x$$

$$\underline{3}$$

$$\boxed{x} \cdot x + 1 \cdot \boxed{x}$$

$$\underline{3 + 1 = 4}$$

$$x(x+1)$$

$$(x+1)(x+3)$$

$$3. \quad \frac{x^2 - 9}{3x + 9} = \frac{(x-3)\cancel{(x+3)}}{3\cancel{(x+3)}} = \frac{x-3}{3}$$

$$x^2 - 9$$

$$3x + 9$$

$$(x-3)(x+3)$$

$$\boxed{3} \cdot x + 3 \cdot \boxed{3}$$

$$3(x+3)$$

$$4. \quad \frac{x^2 + 6x - 7}{x^2 + 12x + 35} = \frac{\cancel{(x+7)}(x-1)}{\cancel{(x+7)}(x+5)} = \frac{x-1}{x+5}$$

$$x^2 + 6x - 7$$

$$x^2 + 12x + 35$$

$$\underline{-7}$$

$$\underline{35}$$

$$\underline{7 - 1 = 6}$$

$$35 + 1 = 36$$

$$\underline{7 + 5 = 12}$$

$$(x+7)(x-1)$$

$$(x+7)(x+5)$$