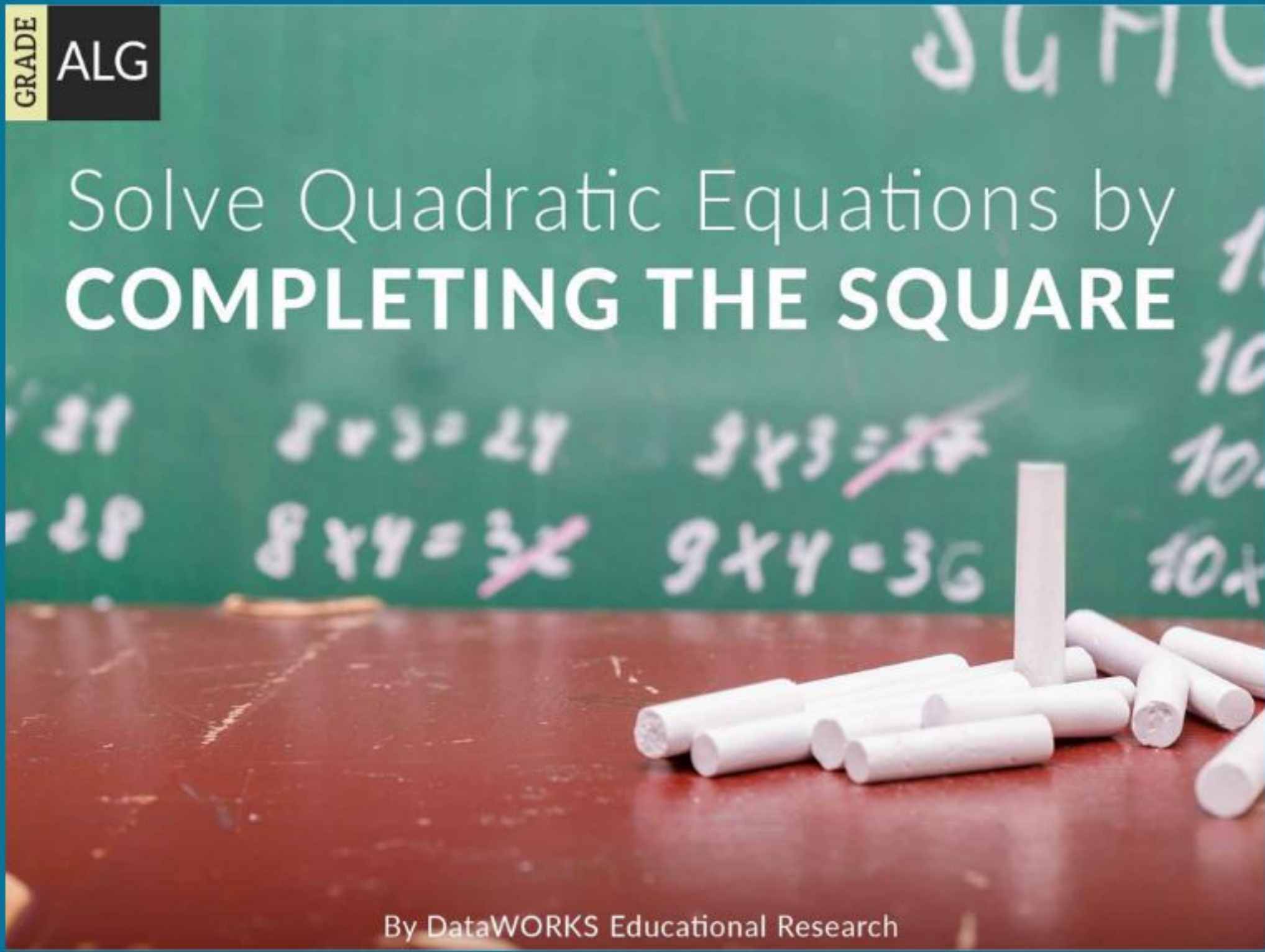
GRADE ALG

Solve Quadratic Equations by **COMPLETING THE SQUARE**



By DataWORKS Educational Research

Learning Objective

We will solve quadratic equations by completing the square.

CFU

What are we going to learn?

Activate Prior Knowledge

1. $x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = 3 \quad x = -3$

2. $x^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = 4 \quad x = -4$

Make Connection

Students, you already know how to take the square root of both sides of an equation. Now, we will solve quadratic equations by completing the square.

A **quadratic equation** is a second-degree polynomial[1] equation.

$x^2 + 2x = 35$ $\quad$ $x^2 - 8x = 9$

Completing the square is a method for solving a quadratic equation by creating a perfect square.

- *This method can be used when the coefficient of the x^2 term is 1.*
- *The solution of a quadratic equation consist of all values (roots) which make the equation true.*
- *Quadratic equations have, at most, 2 solutions.*

Completing the Square:

$x^2 + 2x = 35$

Complete the square by adding the square of half the coefficient[2] of x to both sides. $\left(\frac{b}{2}\right)^2$ $f(x) = ax^2 + bx + c$	$\left(\frac{2}{2}\right)^2 = (1)^2 = 1$ $x^2 + 2x + 1 = 35 + 1$ $x^2 + 2x + 1 = 36$
Convert the left-hand side to a binomial square. Simplify the right-hand side.	$(x+1)^2 = 36$
Take the square root of both sides. Remember to use ±.	$\sqrt{(x+1)^2} = \pm\sqrt{36}$
	$x + 1 = 6$ $\quad$ $x + 1 = -6$
Solve for the solutions of the quadratic equation.	$x = 6 - 1$ $\quad$ $x = -6 - 1$ $x = 5$ $\quad$ $x = -7$

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What must be added to complete the square on the following equations? How do you know?

A. $x^2 + 10x + 25$

$\left(\frac{10}{2}\right)^2 = (5)^2 = 25$

B. $x^2 + 6x + 9$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

In your own words, what is completing the square?

"Completing the square is _____."

Vocabulary

1. (second-degree polynomial) an equation with more than one term in which the highest power on any term is 2
2. number multiplied by the variable

Skill Development/Guided Practice

Completing the square is a method for solving a quadratic equation by creating a perfect square.

- *This method can be used when the coefficient of the x^2 term is 1.*
- *The solution of a quadratic equation consist of all values (roots) which make the equation true.*

Perfect Squares

$(x+1)^2 = (x+1)(x+1) = x^2 + 2x + 1$
$(x+2)^2 = (x+2)(x+2) = x^2 + 4x + 4$
$(x+3)^2 = (x+3)(x+3) = x^2 + 6x + 9$
$(x+7)^2 = (x+7)(x+7) = x^2 + 14x + 49$
$(x-1)^2 = (x-1)(x-1) = x^2 - 2x + 1$
$(x-2)^2 = (x-2)(x-2) = x^2 - 4x + 4$
$(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$

Solve quadratic equations by completing the square.

1. Complete the square by adding the square of half the coefficient of x to both sides.
2. Convert the left-hand side to a binomial square. Simplify the right-hand side.
3. Take the square root of both sides. Remember to use ±.
4. Solve for the solutions to the quadratic equation.

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1. How did I/you complete the square?
2. How did I/you convert the left-hand side to a binomial square?
4. How did I/you solve for the solutions to the quadratic equation?

1.

$x^2 + 14x = 15 \rightarrow \left(\frac{14}{2}\right)^2 = (7)^2 = 49$

$x^2 + 14x + 49 = 15 + 49$

$(x+7)^2 = 64$

$\sqrt{(x+7)^2} = \pm\sqrt{64}$

The solutions to the quadratic equation are x= 1 and x= -15

$x + 7 = 8$ | $x + 7 = -8$

$x = 8 - 7$ | $x = -8 - 7$

$x = 1$ | $x = -15$

2.

$x^2 + 6x = 16 \rightarrow \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$x^2 + 6x + 9 = 16 + 9$

$(x+3)^2 = 25$

$\sqrt{(x+3)^2} = \pm\sqrt{25}$

The solutions to the quadratic equation are x= 2 and x= -8

$x + 3 = 5$ | $x + 3 = -5$

$x = 5 - 3$ | $x = -5 - 3$

$x = 2$ | $x = -8$

Completing the square is a method for solving a quadratic equation by creating a perfect square.

- *This method can be used when the coefficient of the x^2 term is 1.*
- *The solution of a quadratic equation consist of all values (roots) which make the equation true.*

Perfect Squares

$(x+1)^2 = (x+1)(x+1) = x^2 + 2x + 1$
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$(x-2)^2 = (x-2)(x-2) = x^2 - 4x + 4$
$(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$

Solve quadratic equations by completing the square.

1. Complete the square by adding the square of half the coefficient of x to both sides.
2. Convert the left-hand side to a binomial square. Simplify the right-hand side.
3. Take the square root of both sides. Remember to use ±.
4. Solve for the solutions to the quadratic equation.

CFU

1. How did I/you complete the square?
2. How did I/you convert the left-hand side to a binomial square?
4. How did I/you solve for the solutions to the quadratic equation?

3. $x^2 - 8x = 9$

$x^2 - 8x + 16 = 9 + 16$ $\rightarrow \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$(x-4)^2 = 25$

$\sqrt{(x-4)^2} = \pm\sqrt{25}$

The solutions to the quadratic equation are x= 9 and x= -1

$x - 4 = 5$ $\quad$ $x - 4 = -5$

$x = 5 + 4$ $\quad$ $x = -5 + 4$

$x = 9$ $\quad$ $x = -1$

4. $x^2 - 2x = 15$

$x^2 - 2x + 1 = 15 + 1$ $\rightarrow \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$(x-1)^2 = 16$

$\sqrt{(x-1)^2} = \pm\sqrt{16}$

The solutions to the quadratic equation are x= 5 and x= -3

$x - 1 = 4$ $\quad$ $x - 1 = -4$

$x = 4 + 1$ $\quad$ $x = -4 + 1$

$x = 5$ $\quad$ $x = -3$

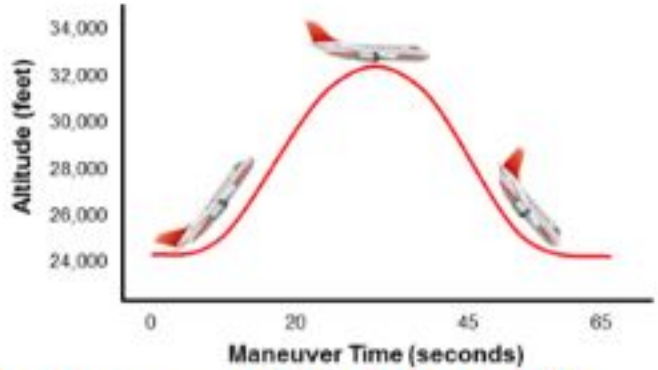
Relevance

Completing the square is a method for solving a quadratic equation by creating a perfect square.

- *This method can be used when the coefficient of the x^2 term is 1.*
- *The solution of a quadratic equation consist of all values (roots) which make the equation true.*

1 *Solving quadratic equations by completing the square will help you graph real-world quadratic equations.*

The "Weightless Wonder" is a plane used by NASA to train astronauts in zero-gravity situations. It accelerates upward and then descends fast enough for the effects of gravity to be negated, producing a feeling of weightlessness.



2 *Solving quadratic equations by completing the square will help you do well on tests.*

Sample Test Question:

32. What quantity should be added to both sides of this equation to complete the square?

$$x^2 - 8x = 5$$

A 4
B -4
C 16
D -16

CFU

Does anyone else have another reason why it is relevant to solve quadratic equations by completing the square? (Pair-Share) Why is it relevant to solve quadratic equations or completing the square? You may give one of my reasons or one of your own. Which reason is more relevant to you? Why?

A **quadratic equation** is a second-degree polynomial equation.
Completing the square is a method for solving a quadratic equation by creating a perfect square.

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$(x-2)^2 = (x-2)(x-2) = x^2 - 4x + 4$
$(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$

Skill Closure

Solve quadratic equations by completing the square.

1. Complete the square by adding the square of half the coefficient of x to both sides.
2. Convert the left-hand side to a binomial square. Simplify the right-hand side.
3. Take the square root of both sides. Remember to use ±.
4. Solve for the solutions to the quadratic equation.

1. $x^2 + 6x = 7$

$\rightarrow \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$x^2 + 6x + 9 = 7 + 9$

$(x+3)^2 = 16$

$\sqrt{(x+3)^2} = \pm\sqrt{16}$

$x + 3 = 4$ | $x + 3 = -4$

$x = 4 - 3$ | $x = -4 - 3$

$x = 1$ | $x = -7$

The solutions to the quadratic equation are x= 1 and x= -7

2. $x^2 - 6x = 7$

$\rightarrow \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$x^2 - 6x + 9 = 7 + 9$

$(x-3)^2 = 16$

$\sqrt{(x-3)^2} = \pm\sqrt{16}$

$x - 3 = 4$ | $x - 3 = -4$

$x = 4 + 3$ | $x = -4 + 3$

$x = 7$ | $x = -1$

The solutions to the quadratic equation are x= 7 and x= -1

Summary Closure

What did you learn today about solving quadratic equations by completing the square?
(Pair-Share)

A **quadratic equation** is a second-degree polynomial equation.
Completing the square is a method for solving a quadratic equation by creating a perfect square.

- *This method can be used when the coefficient of the x^2 term is 1.*
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Solve quadratic equations by completing the square.

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$(x-2)^2 = (x-2)(x-2) = x^2 - 4x + 4$
$(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$

1.

$x^2 + 4x = 12$

$\rightarrow \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$x^2 + 4x + 4 = 12 + 4$

The solutions to the quadratic equation are x= 2 and x= -6

$(x+2)^2 = 16$

$\sqrt{(x+2)^2} = \pm\sqrt{16}$

$x + 2 = 4$ | $x + 2 = -4$

$x = 4 - 2$ | $x = -4 - 2$

$x = 2$ | $x = -6$

2.

$x^2 + 4x = 21$

$\rightarrow \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$x^2 + 4x + 4 = 21 + 4$

The solutions to the quadratic equation are x= 3 and x= -7

$(x+2)^2 = 25$

$\sqrt{(x+2)^2} = \pm\sqrt{25}$

$x + 2 = 5$ | $x + 2 = -5$

$x = 5 - 2$ | $x = -5 - 2$

$x = 3$ | $x = -7$

A quadratic equation is a second-degree polynomial equation.
Completing the square is a method for solving a quadratic equation by creating a perfect square.

- *This method can be used when the coefficient of the x^2 term is 1.*
- *The solution of a quadratic equation consist of all values (roots) which make the equation true.*

Solve quadratic equations by completing the square.

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4. Solve for the solutions to the quadratic equation.

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$(x+7)^2 = (x+7)(x+7) = x^2 + 14x + 49$
$(x-1)^2 = (x-1)(x-1) = x^2 - 2x + 1$
$(x-2)^2 = (x-2)(x-2) = x^2 - 4x + 4$
$(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$

3.

$x^2 - 2x = 8$

$\rightarrow \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$x^2 - 2x + 1 = 8 + 1$

$(x-1)^2 = 9$

$\sqrt{(x-1)^2} = \pm\sqrt{9}$

The solutions to the quadratic equation are x= 4 and x= -2

$x - 1 = 3$
$x = 3 + 1$
$x = 4$

$x - 1 = -3$
$x = -3 + 1$
$x = -2$

4.

$x^2 - 10x = 24$

$\rightarrow \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$x^2 - 10x + 25 = 24 + 25$

$(x-5)^2 = 49$

$\sqrt{(x-5)^2} = \pm\sqrt{49}$

The solutions to the quadratic equation are x= 12 and x= -2

$x - 5 = 7$
$x = 7 + 5$
$x = 12$

$x - 5 = -7$
$x = -7 + 5$
$x = -2$

1.

$x^2 + 8x = 84$

$x^2 + 8x + 16 = 84 + 16$ $\rightarrow \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

$(x+4)^2 = 100$

$\sqrt{(x+4)^2} = \pm\sqrt{100}$

The solutions to the quadratic equation are x= 6 and x= -14

$x + 4 = 10$ $\quad$ $x + 4 = -10$

$x = 10 - 4$ $\quad$ $x = -10 - 4$

$x = 6$ $\quad$ $x = -14$

2.

$x^2 + 2x = 24$

$x^2 + 2x + 1 = 24 + 1$ $\rightarrow \left(\frac{2}{2}\right)^2 = (1)^2 = 1$

$(x+1)^2 = 25$

$\sqrt{(x+1)^2} = \pm\sqrt{25}$

The solutions to the quadratic equation are x= 4 and x= -6

$x + 1 = 5$ $\quad$ $x + 1 = -5$

$x = 5 - 1$ $\quad$ $x = -5 - 1$

$x = 4$ $\quad$ $x = -6$

3.

$x^2 - 2x = 48$

$x^2 - 2x + 1 = 48 + 1$ $\rightarrow \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$(x-1)^2 = 49$

$\sqrt{(x-1)^2} = \pm\sqrt{49}$

The solutions to the quadratic equation are x= 8 and x= -6

$x - 1 = 7$ $\quad$ $x - 1 = -7$

$x = 7 + 1$ $\quad$ $x = -7 + 1$

$x = 8$ $\quad$ $x = -6$

4.

$x^2 - 8x = 0$

$x^2 - 8x + 16 = 0 + 16$ $\rightarrow \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$(x-4)^2 = 16$

$\sqrt{(x-4)^2} = \pm\sqrt{16}$

The solutions to the quadratic equation are x= 8 and x= 0

$x - 4 = 4$ $\quad$ $x - 4 = -4$

$x = 4 + 4$ $\quad$ $x = -4 + 4$

$x = 8$ $\quad$ $x = 0$

1.

$x^2 + 4x = 32$

$\rightarrow \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$x^2 + 4x + 4 = 32 + 4$

$(x+2)^2 = 36$

$\sqrt{(x+2)^2} = \pm\sqrt{36}$

The solutions to the quadratic equation are x= 4 and x= -8

$x + 2 = 6$

$x = 6 - 2$

$x = 4$

$x + 2 = -6$

$x = -6 - 2$

$x = -8$

2.

$x^2 + 12x = 28$

$\rightarrow \left(\frac{12}{2}\right)^2 = (6)^2 = 36$

$x^2 + 12x + 36 = 28 + 36$

$(x+6)^2 = 64$

$\sqrt{(x+6)^2} = \pm\sqrt{64}$

The solutions to the quadratic equation are x= 2 and x= -14

$x + 6 = 8$

$x = 8 - 6$

$x = 2$

$x + 6 = -8$

$x = -8 - 6$

$x = -14$

3.

$x^2 - 2x = 80$

$\rightarrow \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$x^2 - 2x + 1 = 80 + 1$

$(x-1)^2 = 81$

$\sqrt{(x-1)^2} = \pm\sqrt{81}$

The solutions to the quadratic equation are x= 10 and x= -8

$x - 1 = 9$

$x = 9 + 1$

$x = 10$

$x - 1 = -9$

$x = -9 + 1$

$x = -8$

4.

$x^2 - 8x = 9$

$\rightarrow \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$x^2 - 8x + 16 = 9 + 16$

$(x-4)^2 = 25$

$\sqrt{(x-4)^2} = \pm\sqrt{25}$

The solutions to the quadratic equation are x= 9 and x= -1

$x - 4 = 5$

$x = 5 + 4$

$x = 9$

$x - 4 = -5$

$x = -5 + 4$

$x = -1$

1.

$x^2 + 6x = 55$

$\rightarrow \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$x^2 + 6x + 9 = 55 + 9$

$(x+3)^2 = 64$

$\sqrt{(x+3)^2} = \pm\sqrt{64}$

The solutions to the quadratic equation are x= 5 and x= -11

$x + 3 = 8$

$x = 8 - 3$

$x = 5$

$x + 3 = -8$

$x = -8 - 3$

$x = -11$

2.

$x^2 + 4x = 60$

$\rightarrow \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$x^2 + 4x + 4 = 60 + 4$

$(x+2)^2 = 64$

$\sqrt{(x+2)^2} = \pm\sqrt{64}$

The solutions to the quadratic equation are x= 6 and x= -10

$x + 2 = 8$

$x = 8 - 2$

$x = 6$

$x + 2 = -8$

$x = -8 - 2$

$x = -10$

3.

$x^2 - 4x = 21$

$\rightarrow \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$x^2 - 4x + 4 = 21 + 4$

$(x-2)^2 = 25$

$\sqrt{(x-2)^2} = \pm\sqrt{25}$

The solutions to the quadratic equation are x= 7 and x= -3

$x - 2 = 5$

$x = 5 + 2$

$x = 7$

$x - 2 = -5$

$x = -5 + 2$

$x = -3$

4.

$x^2 - 10x = 24$

$\rightarrow \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$x^2 - 10x + 25 = 24 + 25$

$(x-5)^2 = 49$

$\sqrt{(x-5)^2} = \pm\sqrt{49}$

The solutions to the quadratic equation are x= 12 and x= -2

$x - 5 = 7$

$x = 7 + 5$

$x = 12$

$x - 5 = -7$

$x = -7 + 5$

$x = -2$