GRADE ALG

Solve Quadratic Equations Using the Quadratic Formula

By DataWORKS Educational Research

Learning Objective

We will solve quadratic equations using the quadratic formula.

CFU

What are we going to learn?

Activate Prior Knowledge

Substitute to evaluate the expression $b^2 - 4ac$.

1. $a = 2, b = 5, c = -3$

$b^2 - 4ac$

$(5)^2 - 4(2)(-3)$

$25 + 24$

49

2. $a = 2, b = 3, c = -1$

$b^2 - 4ac$

$(3)^2 - 4(2)(-1)$

$9 + 8$

17

Multiplying with Negatives

$(-)(-) = +$

$(+)(-) = -$

Make Connection

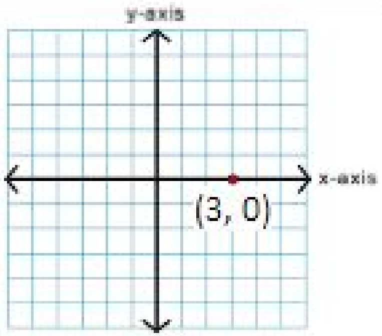
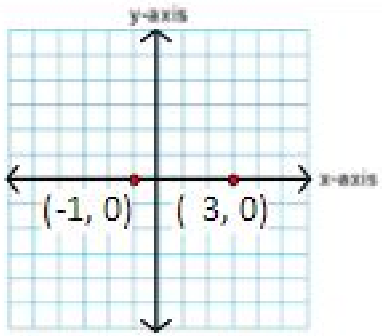
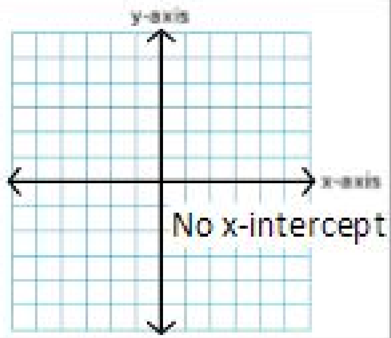
Students, you already know how to substitute and evaluate expressions. Now, we will use this skill while using the quadratic formula to solve quadratic equations.

A **quadratic equation** is a second-degree polynomial equation.

$$x^2 + 2x = 35 \qquad x^2 - 8x = 9$$

The **solution of a quadratic equation** is the set of x-values that make the equation true.

- *The solution of a quadratic equation consists of all x-intercepts (roots).*
- *Quadratic equations have 1, 2 or no real solutions.*

$x^2 - 6x + 9 = 0$	$x^2 - 2x - 3 = 0$	$x^2 + x + 1 = 0$
Solution Set: { 3 }	Solution Set: { -1,3 }	Solution Set: {not real}
		

CFU

Which quadratic equation below has two solutions? How do you know?

Which quadratic equation below has no solutions? How do you know?

What is the solution set for the quadratic equation that created graph A? How do you know?

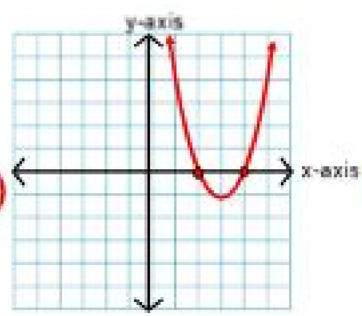
A {2}

B {2, 4}

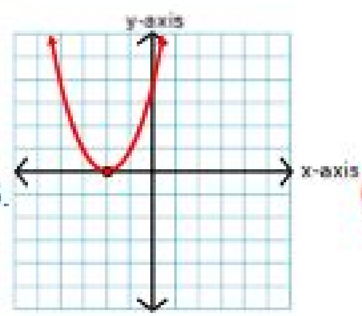
In your own words, what is the solution of a quadratic equation?

"The solution of a quadratic equation is __________."

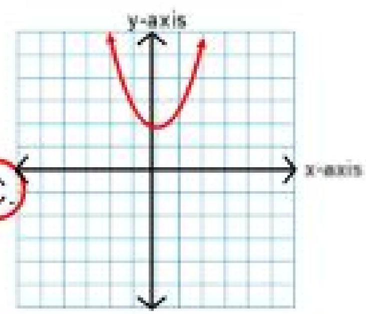
A.



B.



C.



The **quadratic formula** is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Using the quadratic formula

1 solution

$$x^2 - 6x + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1,\ b = -6,\ c = 9$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 36}}{2}$$

$$x = \frac{6 \pm \sqrt{0}}{2}$$

$$x = \frac{6}{2}$$

$$x = 3$$

The solution set is {3}.

2 solutions

$$x^2 - 2x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1,\ b = -2,\ c = -3$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$x = \frac{2 \pm 4}{2}$$

$$x = \frac{6}{2}, \frac{-2}{2}$$

$$x = 3, -1$$

The solution set is {3, -1}.

No real solutions

$$x^2 + x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1,\ b = 1,\ c = 1$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

The square root of -3 is not a real number so there are no real solutions.

CFU 2

Identify a, b, and c for

$2x^2 + 4x - 16 = 0$

How do you know?

A a = 2, b = -16, c = 4

(B) a = 2, b = 4, c = -16

Which equation correctly shows

$2x^2 + 4x - 16 = 0$

placed in the quadratic formula? How do you know?

A $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-16)}}{2(1)}$

(B) $x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-16)}}{2(2)}$

In your own words, what is the quadratic formula?

"The quadratic formula is__________."

Skill Development/Guided Practice

The **quadratic formula** is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

CFU

2. How did I/you identify a, b, and c?
3. How did I/you simplify the equation?
4. How did I/you write two equations to solve for x?
5. How did I/you solve the two equations for x?

1. $2x^2 - 12x - 14 = 0$

$a = 2,\ b = -12,\ c = -14$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(-14)}}{2(2)}$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{4}$$

$$x = \frac{12 \pm \sqrt{256}}{4}$$

$$x = \frac{12 \pm 16}{4}$$

$$x = \frac{12 + 16}{4} \qquad x = \frac{12 - 16}{4}$$

$$x = \frac{28}{4} \quad x = 7 \qquad x = \frac{-4}{4} \quad x = -1$$

The solution set of the quadratic equation is {7, -1}.

2. $2x^2 - 10x + 8 = 0$

$a = 2,\ b = -10,\ c = 8$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(8)}}{2(2)}$$

$$x = \frac{10 \pm \sqrt{100 - 64}}{4}$$

$$x = \frac{10 \pm \sqrt{36}}{4}$$

$$x = \frac{10 \pm 6}{4}$$

$$x = \frac{10 + 6}{4} \qquad x = \frac{10 - 6}{4}$$

$$x = \frac{16}{4} \quad x = 4 \qquad x = \frac{4}{4} \quad x = 1$$

The solution set of the quadratic equation is {4,1}.

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

CFU

2. How did I/you identify a, b, and c?
3. How did I/you simplify the equation?
4. How did I/you write two equations to solve for x?
5. How did I/you solve the two equations for x?

3\. $x^2 + 3x + 1 = 0$

$a = 1,\ b = 3,\ c = 1$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

$$x = \frac{-3 + \sqrt{5}}{2} \qquad x = \frac{-3 - \sqrt{5}}{2}$$

The solution set of the quadratic equation is $\left\{\frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}\right\}$ $\approx \{-0.38,\ -2.62\}$

4\. $x^2 + 5x + 5 = 0$

$a = 1,\ b = 5,\ c = 5$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 20}}{2}$$

$$x = \frac{-5 \pm \sqrt{5}}{2}$$

$$x = \frac{-5 + \sqrt{5}}{2} \qquad x = \frac{-5 - \sqrt{5}}{2}$$

The solution set of the quadratic equation is $\left\{\frac{-5+\sqrt{5}}{2}, \frac{-5-\sqrt{5}}{2}\right\}$ $\approx \{-1.38,\ -3.62\}$

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

CFU

2. How did I/you identify a, b, and c?
3. How did I/you simplify the equation?
4. How did I/you write two equations to solve for x?
5. How did I/you solve the two equations for x?

5. $x^2 - 6x + 9 = 0$

$a = 1,\ b = -6,\ c = 9$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 36}}{2}$$

$$x = \frac{6 \pm \sqrt{0}}{2}$$

$$x = \frac{6 \pm 0}{2}$$

$$x = \frac{6+0}{2} \qquad x = \frac{6}{2} \qquad x = 3$$

$$x = \frac{6-0}{2} \qquad x = \frac{6}{2} \qquad x = 3$$

The solution set of the quadratic equation is {3}.

6. $x^2 - 8x + 16 = 0$

$a = 1,\ b = -8,\ c = 16$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 64}}{2}$$

$$x = \frac{8 \pm \sqrt{0}}{2}$$

$$x = \frac{8 \pm 0}{2}$$

$$x = \frac{8+0}{2} \qquad x = \frac{8}{2} \qquad x = 4$$

$$x = \frac{8-0}{2} \qquad x = \frac{8}{2} \qquad x = 4$$

The solution set of the quadratic equation is {4}.

The **quadratic formula** is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

CFU

2. How did I/you identify a, b, and c?
3. How did I/you simplify the equation?
4. How did I/you write two equations to solve for x?
5. How did I/you solve the two equations for x?

7. $x^2 + 3x + 4 = 0$

$a = 1,\ b = 3,\ c = 4$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 16}}{2}$$

$$x = \frac{-3 \pm \sqrt{-7}}{2}$$

There are no real solutions for the square root of -7, so there are no real solutions to the quadratic equation.

8. $x^2 + 4x + 5 = 0$

$a = 1,\ b = 4,\ c = 5$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$x = \frac{-4 \pm \sqrt{-4}}{2}$$

There are no real solutions for the square root of -4, so there are no real solutions to the quadratic equation.

Relevance

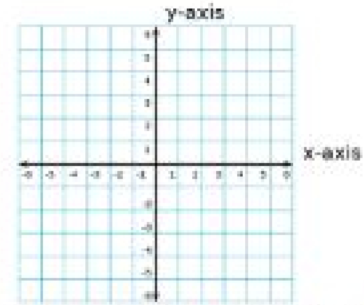
The <u>quadratic formula</u> is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

1 *Solving quadratic equations using the quadratic formula will help you solve real-world problems.*

2 *Solving quadratic equations using the quadratic formula will help you it find precise x-intercept values.*

$12x^2 + x - 1 = 0$



It is difficult to tell precisely where the x-intercepts are. The quadratic formula will give precise roots.

Too shallow, not enough height

Too much height, not enough distance

Perfect Kick!

3 *Solving quadratic equations will help you do well on tests.*

Sample Test Question:

32. What is the solution set of the quadratic equation $8x^2 + 2x + 1 = 0$?

A $\left\{-\frac{1}{2}, \frac{1}{4}\right\}$

B $\{-1+\sqrt{2}, -1\ \sqrt{2}\}$

C $\left\{\frac{-1+\sqrt{7}}{8}, \frac{-1-\sqrt{7}}{8}\right\}$

CFU

Does anyone else have another reason why it is relevant to solve quadratic equations using the quadratic formula? (Pair-Share) Why is it relevant to solve quadratic equations using the quadratic formula? You may give one of my reasons or one of your own. Which reason is more relevant to you? Why?

DataWORKS Educational Research

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Skill Closure

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

Word Bank

- quadratic equation
- formula

1. $2x^2 - 2x - 12 = 0$

$a = 2,\ b = -2,\ c = -12$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-12)}}{2(2)}$$

$$x = \frac{2 \pm \sqrt{4 + 96}}{4}$$

$$x = \frac{2 \pm \sqrt{100}}{4}$$

$$x = \frac{2 \pm 10}{4}$$

$$x = \frac{2 + 10}{4} \qquad x = \frac{2 - 10}{4}$$

$$x = \frac{12}{4} \quad x = 3 \qquad x = \frac{-8}{4} \quad x = -2$$

The solution set of the quadratic equation is {3, -2}.

2. $x^2 - 10x + 25 = 0$

$a = 1,\ b = -10,\ c = 25$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 100}}{2}$$

$$x = \frac{10 \pm \sqrt{0}}{2}$$

$$x = \frac{10 \pm 0}{2}$$

$$x = \frac{10 + 0}{2} \qquad x = \frac{10 - 0}{2}$$

$$x = \frac{10}{2} \quad x = 5 \qquad x = \frac{10}{2} \quad x = 5$$

The solution set of the quadratic equation is {5}.

Summary Closure

What did you learn today about solving quadratic equations using the quadratic formula?
(Pair-Share) Use words from the word bank.

Independent Practice

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$ax^2 + bx + c = 0$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

1. $x^2 + x - 12 = 0$

$a = 1, b = 1, c = -12$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2}$$

$$x = \frac{-1 \pm \sqrt{49}}{2}$$

$$x = \frac{-1 \pm 7}{2}$$

$$x = \frac{-1 + 7}{2} \qquad x = \frac{-1 - 7}{2}$$

$$x = \frac{6}{2} \quad x = 3 \qquad x = \frac{-8}{2} \quad x = -4$$

The solutions to the quadratic equation are x = 3 and x = -4.

2. $x^2 + 6x + 9 = 0$

$a = 1, b = 6, c = 9$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 36}}{2}$$

$$x = \frac{-6 \pm \sqrt{0}}{2}$$

$$x = \frac{-6 \pm 0}{2}$$

$$x = \frac{-6 + 0}{2} \qquad x = \frac{-6 - 0}{2}$$

$$x = \frac{-6}{2} \quad x = -3 \qquad x = \frac{-6}{2} \quad x = -3$$

The solution to the quadratic equation is x = -3.

The <u>quadratic formula</u> is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

3. $2x^2 - x - 1 = 0$

$a = 2, b = -1, c = -1$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1+8}}{4}$$

$$x = \frac{1 \pm \sqrt{9}}{4}$$

$$x = \frac{1 \pm 3}{4}$$

$x = \frac{1+3}{4}$ $\quad$ $x = \frac{1-3}{4}$

$x = \frac{4}{4}$ $\quad x = 1$ $\quad$ $x = \frac{-2}{4}$ $\quad x = -\frac{1}{2}$

The solutions to the quadratic equation are $x = 1$ and $x = -\frac{1}{2}$.

4. $4x^2 + 4x + 1 = 0$

$a = 4, b = 4, c = 1$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16 - 16}}{8}$$

$$x = \frac{-4 \pm \sqrt{0}}{8}$$

$$x = \frac{-4 \pm 0}{8}$$

$x = \frac{-4+0}{8}$ $\quad$ $x = \frac{-4-0}{8}$

$x = \frac{-4}{8}$ $\quad x = -\frac{1}{2}$ $\quad$ $x = \frac{-4}{8}$ $\quad x = -\frac{1}{2}$

The solution to the quadratic equation is $x = -\frac{1}{2}$.

The quadratic formula is a method for solving a quadratic equation.

- *... standard form, the a, b, and c ... substituted into the quadratic formula to find all the x-values which make the equation true.*

Snipping Tool

Drag the cursor around the area you want to capture.

Standard Form

$ax^2 + bx + c = 0$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the **quadratic formula.**

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

1. $2x^2 + 5x - 7 = 0$

$a = 2,\ b = 5,\ c = -7$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 + 56}}{4}$$

$$x = \frac{-5 \pm \sqrt{81}}{4}$$

$$x = \frac{-5 \pm 9}{4}$$

$$x = \frac{-5 + 9}{4} \qquad x = \frac{-5 - 9}{4}$$

$$x = \frac{4}{4} \quad x = 1 \qquad x = \frac{-14}{4} \quad x = \frac{-7}{2}$$

The solutions to the quadratic equation are x = 1 and $x = -\frac{7}{2}$.

2. $2x^2 - 9x + 4 = 0$

$a = 2,\ b = -9,\ c = 4$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{9 \pm \sqrt{81 - 32}}{4}$$

$$x = \frac{9 \pm \sqrt{49}}{4}$$

$$x = \frac{9 \pm 7}{4}$$

$$x = \frac{9 + 7}{4} \qquad x = \frac{9 - 7}{4}$$

$$x = \frac{16}{4} \quad x = 4 \qquad x = \frac{2}{4} \quad x = \frac{1}{2}$$

The solutions to the quadratic equation are x = 4 and $x = \frac{1}{2}$.

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$ax^2 + bx + c = 0$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

3. $3x^2 - 2x - 8 = 0$

$a = 3,\ b = -2,\ c = -8$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-8)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 + 96}}{6}$$

$$x = \frac{2 \pm \sqrt{100}}{6}$$

$$x = \frac{2 \pm 10}{6}$$

$$x = \frac{2 + 10}{6} \qquad x = \frac{2 - 10}{6}$$

$$x = \frac{12}{6} \quad x = 2 \qquad x = \frac{-8}{6} \quad x = -1\frac{1}{3}$$

The solutions to the quadratic equation are x = 2 and $x = -1\frac{1}{3}$.

4. $2x^2 + 6x + 4 = 0$

$a = 2,\ b = 6,\ c = 4$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 - 32}}{4}$$

$$x = \frac{-6 \pm \sqrt{4}}{4}$$

$$x = \frac{-6 \pm 2}{4}$$

$$x = \frac{-6 + 2}{4} \qquad x = \frac{-6 - 2}{4}$$

$$x = \frac{-4}{4} \quad x = -1 \qquad x = \frac{-8}{4} \quad x = -2$$

The solutions to the quadratic equation are x = -1 and x = -2.

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

1. $4x^2 - 4x - 3 = 0$

$a = 4,\ b = -4,\ c = -3$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{4 \pm \sqrt{16 + 48}}{8}$$

$$x = \frac{4 \pm \sqrt{64}}{8}$$

$$x = \frac{4 \pm 8}{8}$$

$$x = \frac{4+8}{8} \qquad x = \frac{4-8}{8}$$

$$x = \frac{12}{8} \quad x = \frac{3}{2} \qquad x = \frac{-4}{8} \quad x = -\frac{1}{2}$$

The solutions to the quadratic equation are $x = \frac{3}{2}$ and $x = -\frac{1}{2}$.

2. $x^2 + 4x + 3 = 0$

$a = 1,\ b = 4,\ c = 3$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 12}}{2}$$

$$x = \frac{-4 \pm \sqrt{4}}{2}$$

$$x = \frac{-4 \pm 2}{2}$$

$$x = \frac{-4+2}{2} \qquad x = \frac{-4-2}{2}$$

$$x = \frac{-2}{2} \quad x = -1 \qquad x = \frac{-6}{2} \quad x = -3$$

The solutions to the quadratic equation are x = -1 and x = -3.

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

3. $x^2 - 2x - 8 = 0$

$a = 1, b = -2, c = -8$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 32}}{2}$$

$$x = \frac{2 \pm \sqrt{36}}{2}$$

$$x = \frac{2 \pm 6}{2}$$

$$x = \frac{2+6}{4} \qquad x = \frac{2-6}{2}$$

$$x = \frac{8}{2} \quad x = 4 \qquad x = \frac{-4}{2} \quad x = -2$$

The solutions to the quadratic equation are x = 4 and x = -2.

4. $2x^2 + 7x + 3 = 0$

$a = 2, b = 7, c = 3$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49 - 24}}{4}$$

$$x = \frac{-7 \pm \sqrt{25}}{4}$$

$$x = \frac{-7 \pm 5}{4}$$

$$x = \frac{-7+5}{4} \qquad x = \frac{-7-5}{4}$$

$$x = \frac{-2}{4} \quad x = -\frac{1}{2} \qquad x = \frac{-12}{4} \quad x = -3$$

The solutions to the quadratic equation are $x = -\frac{1}{2}$ and x = -3.

The quadratic formula is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Standard Form

$ax^2 + bx + c = 0$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

1. $3x^2 - 4x - 20 = 0$

$a = 3,\ b = -4,\ c = -20$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-20)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{16 + 240}}{6}$$

$$x = \frac{4 \pm \sqrt{256}}{6}$$

$$x = \frac{4 \pm 16}{6}$$

$$x = \frac{4 + 16}{6} \qquad x = \frac{4 - 16}{6}$$

$$x = \frac{20}{6} \quad x = 3\frac{1}{3} \qquad x = \frac{-12}{6} \quad x = -2$$

The solutions to the quadratic equation are $x = 3\frac{1}{3}$ and $x = -2$.

2. $x^2 - 10x + 25 = 0$

$a = 1,\ b = -10,\ c = 25$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 100}}{2}$$

$$x = \frac{10 \pm \sqrt{0}}{2}$$

$$x = \frac{10 \pm 0}{2}$$

$$x = \frac{10 + 0}{2} \qquad x = \frac{10 - 0}{2}$$

$$x = \frac{10}{2} \quad x = 5 \qquad x = \frac{10}{2} \quad x = 5$$

The solution to the quadratic equation is $x = 5$.

Standard Form
$ax^2 + bx + c = 0$

Quadratic Formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The <u>quadratic formula</u> is a method for solving a quadratic equation.

- *In standard form, the a, b, and c values can be substituted into the quadratic formula to find all the x-values which make the equation true.*

Solve quadratic equations using the quadratic formula.

1. Identify a, b, and c.
2. Insert a, b, and c into the quadratic formula.
3. Simplify the equation.
4. Write two equations using both a + and – in place of the ±.
5. Solve the two equations for *x*.

3. $-x^2 + 81 = 0$

$a = -1, b = 0, c = 81$

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(-1)(81)}}{2(-1)}$$

$$x = \frac{0 \pm \sqrt{0 + 324}}{-2}$$

$$x = \frac{0 \pm \sqrt{324}}{-2}$$

$$x = \frac{0 \pm 18}{-2}$$

$$x = \frac{0 + 18}{-2} \qquad x = \frac{0 - 18}{-2}$$

$$x = \frac{18}{-2} \quad x = -9 \qquad x = \frac{-18}{-2} \quad x = 9$$

The solutions to the quadratic equation are x = -9 and x = 9.

4. $x^2 + 7x + 12$

$a = 1, b = 7, c = 12$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 - 48}}{2}$$

$$x = \frac{-7 \pm \sqrt{1}}{2}$$

$$x = \frac{-7 \pm 1}{2}$$

$$x = \frac{-7 + 1}{2} \qquad x = \frac{-7 - 1}{2}$$

$$x = \frac{-6}{2} \quad x = -3 \qquad x = \frac{-8}{2} \quad x = -4$$

The solutions to the quadratic equation are x = -3 and x = -4.

Proof of the Quadratic Formula by completing the square.

Begin with the standard form, which represents any quadratic equation.	$ax^2 + bx + c = 0 \quad (1)$
We need the coefficient[1] of the x^2 term to be one. To do this, we will divide both sides of equation (1) by a. The result is equation (2)	$\frac{ax^2 + bx + c}{a} = \frac{0}{a}$ $\frac{ax^2}{a} + \frac{b}{a}x + \frac{c}{a} = \frac{0}{a}$ $x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad (2)$
Isolate the constant $\frac{c}{a}$ term on the right-hand side of the equation. The result is equation (3).	$x^2 + \frac{b}{a}x + \frac{c}{a} - \frac{c}{a} = 0 - \frac{c}{a}$ $x^2 + \frac{b}{a}x = -\frac{c}{a} \quad (3)$

Complete the Square

$$x^2 + 14x = 15 \rightarrow \left(\frac{14}{2}\right)^2 = (7)^2 = 49$$

$$x^2 + 14x + 49 = 15 + 49$$

$$(x + 7)^2 = 64$$

Notice that the term inside the parentheses before we square is the term that is used in the binomial. This is always true and will be used in the next step of this proof.

Vocabulary

[1] the number in a term, typically in front of a variable

Proof of the Quadratic Formula by completing the square.

Recall the formula for completing the square, where the box represents the coefficient of the *x* term. $\left(\frac{\square}{2}\right)^2$ In our proof, the coefficient of the *x* term is $\frac{b}{a}$, so we will substitute this into the box. This is the term we add to both sides of equation (3), to complete the square. The result is equation (4).	$\left(\frac{\square}{2}\right)^2 = \left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{\frac{b}{a}\cdot\frac{1}{2}}{2\cdot\frac{1}{2}}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{2^2a^2} = \frac{b^2}{4a^2}$ $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$ (4)
Factor the left-hand side of equation (4), the result is equation (5). The example with '7' in "complete the square" on the previous page shows the pattern used here.	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$ (5)
Simplify the right-hand side of equation (5) The result is equation (6)	$\frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{c}{a}\cdot\frac{4a}{4a} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2 - 4ac}{4a^2}$ $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$ (6)
Square Root both sides of equation (6) The result is equation (7)	$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$ $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ (7)
Solve equation (7) for *x* and we will arrive at the Quadratic Formula.	$x + \frac{b}{2a} - \frac{b}{2a} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$