

MAKING & USING A STUDY GUIDE for a test

Exam-5: Using Factors to Solve Quadratic Equations

Study Guide: helps you ① summarize,
② visualize, and analyze ③
concepts learned in class

* Warning: simply making a study guide
does not guarantee you an
A+ on the test.

1

How can you use factoring to solve quadratic equations in standard form?

You can the right side of the quadratic equation. Then set each linear factor equal to . Solve each linear equation. These the solutions of the original quadratic equation.

Just an example:

Solve the polynomial equation by factoring.

$$4x^6 + 4x^5 - 24x^4 = 0$$

$$4x^4(x^2 + x - 6) = 0 \quad \text{Factor out the GCF, } 4x^4.$$

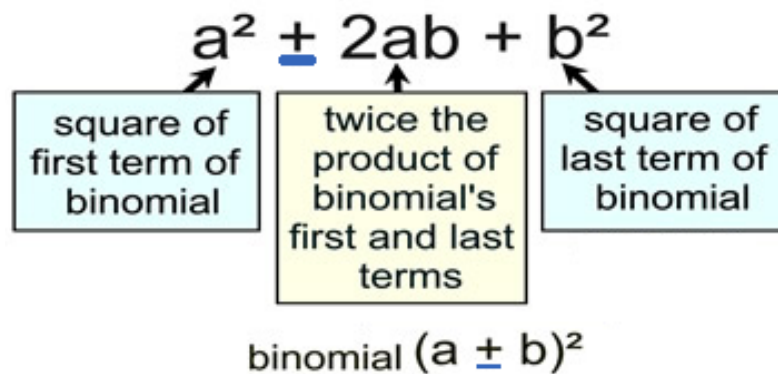
$$4x^4(x + 3)(x - 2) = 0 \quad \text{Factor the quadratic.}$$

$$4x^4 = 0 \text{ or } (x + 3) = 0 \text{ or } (x - 2) = 0 \quad \text{Set each factor equal to 0.}$$

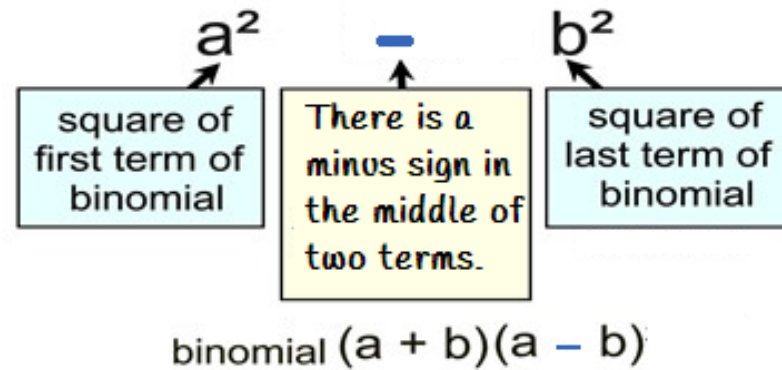
$$x = 0, x = -3, x = 2 \quad \text{Solve for } x.$$

The roots are 0, -3, and 2.

2 Perfect Square Trinomial



Difference of Squares



Identify the expression as a perfect-square trinomial, a difference of squares, or neither. Factor the expression.

$$\sqrt{25p^2} + 80p + \sqrt{64} \quad \text{*Take the square of root of first and last terms}$$

The expression is .

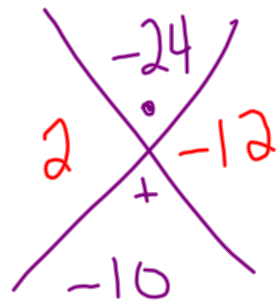
The factored expression is .

- 3 Identify the expression as a perfect-square trinomial, a difference of squares, or neither. Factor the expression.

$$p^2 - 10p - 24$$

The expression is neither

XMETHOD



$$(p+2)(p-12)$$

The factored expression is $(p+2)(p-12)$

Factors of -24	Sum of Factors $+$
1 and -24	$1 - 24 = -23$
2 and -12	$2 - 12 = -10$
3 and -8	$3 - 8 = -5$
4 and -6	$4 - 6 = -2$
-1 and 24	$-1 + 24 = 23$
-2 and 12	$-2 + 12 = 10$
-3 and 8	$-3 + 8 = 5$
-4 and 6	$-4 + 6 = 2$

4 Factor the expressions.

$$x^2 - 14x + 48 = (x - 6)(x - 8)$$

XMETHOD

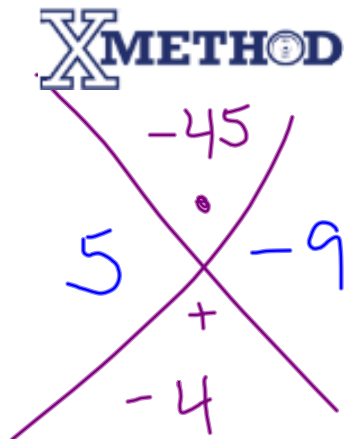
Handwritten diagram for the X-method. It shows a large 'X' with '48' at the top, '-6' on the left, and '-8' on the right. A small '6' is written above the '+' sign, and '-14' is written below it. Red lines cross through the diagram.

Factors of 48	Sums of Factors
-1 and -48	$-1 - 48 = -49$
-2 and -24	$-2 - 24 = -26$
-3 and -16	$-3 - 16 = -19$
-4 and -12	$-4 - 12 = -16$
-6 and -8	$-6 - 8 = -14$

5

Factor the expressions.

$$x^2 - 4x - 45 = (x + 5)(x - 9)$$



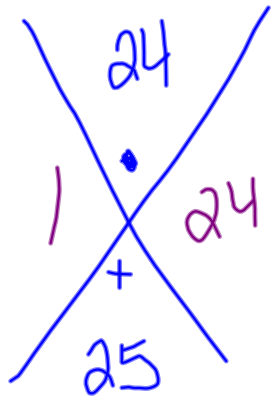
Factors of 45	Sums of Factors
1 and -45	$1 - 45 = -44$
3 and -15	$3 - 15 = -12$
5 and -9	$5 - 9 = -4$
-1 and 45	$-1 + 45 = 44$
-3 and 15	$-3 + 15 = 12$
-5 and 9	$-5 + 9 = 4$

6

Factor the expressions.

$$x^2 + 25x + 24 = (x + 1)(x + 24)$$

XMETHOD



Factors of 24	Sums of Factors
1 and 24	$1 + 24 = 25$
2 and 12	$2 + 12 = 14$
3 and 8	$3 + 8 = 11$
4 and 6	$4 + 6 = 10$

7

Solve the equation.

$$x^2 - 28x + 27 = 0$$

XMETHOD

~~$$\begin{array}{ccc} & 27 & \\ -1 & \cdot & -27 \\ & + & \\ & -28 & \end{array}$$~~



Factors of 27	Sums of Factors
-1 and -27	$-1 - 27 = -28$
-3 and -9	$-3 - 9 = -12$

Factor the equation.

$$(x-1)(x-27) = 0$$

Find the zeroes.

$$\begin{array}{r} x-1=0 \\ +1 \quad +1 \\ \hline x=1 \end{array}$$

$$\begin{array}{r} x-27=0 \\ +27 \quad +27 \\ \hline x=27 \end{array}$$

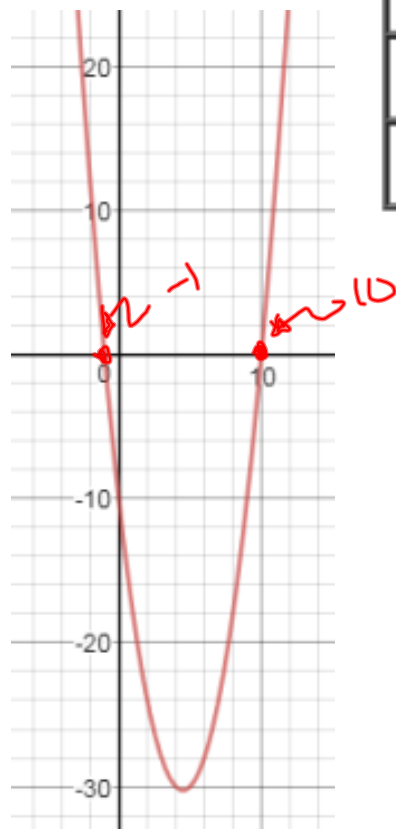
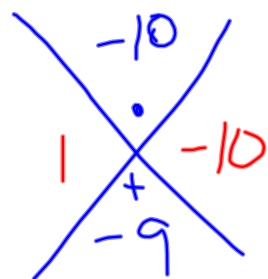
The solutions to the equation are 1 and 27.

8

Solve the equation.

$$x^2 - 9x - 10 = 0$$

XMETHOD



Factors of 10	Sums of Factors
1 and -10	$1 - 10 = -9$
2 and -5	$2 - 5 = -3$
-1 and 10	$-1 + 10 = 9$
-2 and 5	$-2 + 5 = 3$

Factor the equation.

$$(x+1)(x-10) = 0$$

Find the zeroes.

$$\begin{array}{r} x+1=0 \\ -1 \quad -1 \\ \hline x = -1 \end{array}$$

$$\begin{array}{r} x-10=0 \\ +10 \quad +10 \\ \hline x = 10 \end{array}$$

The solutions to the equation are -1 and 10 .

9

Solve the equation.

$$x^2 + 22x = 135$$

$$x^2 + 22x - 135 = 0$$

XMETHOD

$$\begin{array}{ccc} & -135 & \\ -5 & \cdot & 27 \\ & + & \\ & 22 & \end{array}$$



opposite sign

Factors of 135	Sums of Factors
1 and -135	$1 - 135 = -134$
3 and -45	$3 - 45 = -42$
5 and -27	$5 - 27 = -22$
9 and -15	$9 - 15 = -6$
-1 and 135	$-1 + 135 = 134$
-3 and 45	$-3 + 45 = 42$
-5 and 27	$-5 + 27 = 22$
-9 and 15	$-9 + 15 = 6$

Factor the equation.

$$(x-5)(x+27) = 0$$

Find the zeroes.

$$\begin{array}{r} x-5=0 \\ +5 \quad +5 \\ \hline x=5 \end{array} \quad \begin{array}{r} x+27=0 \\ -27 \quad -27 \\ \hline x=-27 \end{array}$$

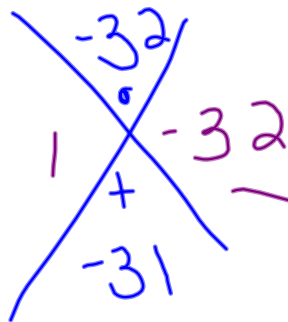
The solutions to the equation are 5 and -27.

10 Solve the equation.

$$x^2 - 31x = 32$$

$$x^2 - 31x - 32 = 0$$

XMETHOD



opposite sign

Factors of 32	Sums of Factors
1 and -32	1 - 32 = -31
2 and -16	2 - 16 = -14
4 and -8	4 - 8 = -4
-1 and 32	-1 + 32 = 31
-2 and 16	-2 + 16 = 14
-4 and 8	-4 + 8 = 4

The solutions to the equation are

and

Drag and drop each pair of solutions next to the equation having those solutions.

11

$$x^2 - 3x - 18 = 0$$

XMETHOD *opposite sign*

-3 and 6



switch
Sign

$$x^2 + 11x + 18 = 0$$

-2 and -9



switch
Sign

$$x^2 + 3x - 18 = 0$$

3 and -6



switch
Sign

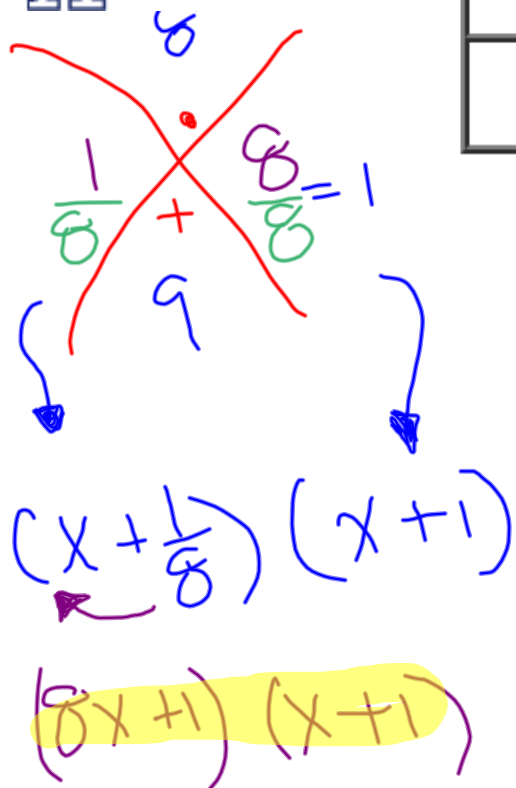
$$x^2 - 11x + 18 = 0$$

2 and 9

12 Factor the quadratic by checking factor pairs.

$$8x^2 + 9x + 1$$

XMETHOD



Factors of a	Factors of c	Outer + Inner
1 and 8	1 and 1	9
2 and 4	1 and 1	6

13

Factor the quadratic by checking factor pairs.

$$9x^2 + 33x + 30$$

$$\underline{3} \underline{9}x^2 + \underline{3} \underline{33}x + \underline{3} \underline{30} = 3(\quad)(\quad)$$

X METHOD

$$\Downarrow$$

$$3x^2 + 11x + 10 \Rightarrow$$

30

5
3

+
=

6
3 = 2

(x + 5/3)(x + 2)

(3x + 5)(x + 2)

$$9x^2 + 33x + 30 = 3(x + 2)(3x + 5)$$

14

Factor the quadratic by checking factor pairs.

$$4x^2 - 7x + 3$$

$$4x^2 - 7x + 3 = (\quad)(\quad)$$

XMETHOD



$$(x-1)\left(x-\frac{3}{4}\right)$$

$$(x-1)(4x-3)$$

15

Factor the quadratic by checking factor pairs.

$$3x^2 - 4x - 7$$

$$3x^2 - 4x - 7 = (\quad)(\quad)$$

X METHOD

$$1 = \frac{3}{3} + \frac{-7}{3}$$

-21
-4



$$(x+1)(x-\frac{7}{3})$$

$$(x+1)(3x-7)$$

$$3x^2 - 4x - 7 = (x+1)(3x-7)$$

16 Solve the following quadratic equation.
 $5x^2 + 21x + 4 = 0$


XMETHOD

~~$\frac{1}{5}$ $\frac{20}{5} = 4$~~

$\frac{1}{5} + \frac{20}{5} = 4$

Switch Sign

$x = \boxed{-\frac{1}{5}}$ or $x = \boxed{-4}$



17

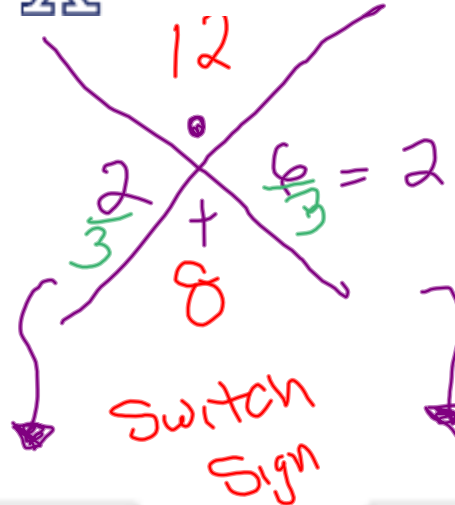
Solve the following quadratic equation.

$$6x^2 + 18x - 2 = 2x - 10$$

$$\begin{array}{r|l} -2x + 10 & -2x + 10 \\ \hline \frac{6x^2}{2} + \frac{16x}{2} + \frac{8}{2} & 0 \end{array}$$

$$3x^2 + 8x + 4 = 0$$

X METHOD



$$x = \boxed{-\frac{2}{3}} \quad \text{or} \quad x = \boxed{-2}$$



18 Solve the following quadratic equation. Simplify the fraction if necessary. **$(8x + 4)(x + 1) = 12$**

	$8x$	4
x	$8x^2$	$4x$
1	$8x$	4

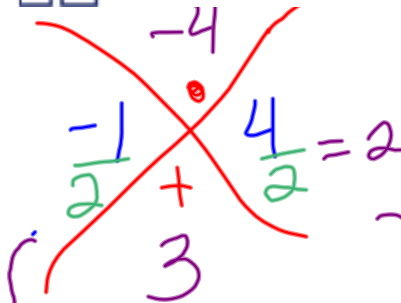
$$8x^2 + 12x + 4 = 12$$

-12 -12

$$\frac{8x^2}{4} + \frac{12x}{4} - \frac{8}{4} = 0$$

$$2x^2 + 3x - 2 = 0$$

XMETHOD



$$x = \boxed{} \frac{1}{2} \quad \text{or} \quad x = \boxed{-2}$$



Switch signs on factors

Solve the following quadratic equation.

19

$$3(2x - 1)(2x + 3) = 24x$$

	2x	-1
2x	4x ²	-2x
3	6x	-3

$$3(4x^2 + 4x - 3) = 24x$$

$$12x^2 + 12x - 9 = 24x$$

-24x -24x

$$\frac{12x^2}{3} - \frac{12x}{3} - \frac{9}{3} = 0$$

$$4x^2 - 4x - 3 = 0$$

XMETHOD

$$\frac{1}{2} = \frac{2}{4} + \frac{-6}{4} = \frac{-4}{4}$$

$$x = \boxed{-\frac{1}{2}} \quad \text{or} \quad x = \boxed{\frac{3}{2}}$$



Switch signs

Factor the perfect-square trinomial.

20

$$2y^3 + 12y^2 + 18y$$

The factored form of $\frac{2y^3}{2y} + \frac{12y^2}{2y} + \frac{18y}{2y}$ is $2y(y+3)^2$.

$$2y(y^2 + 6y + 9)$$

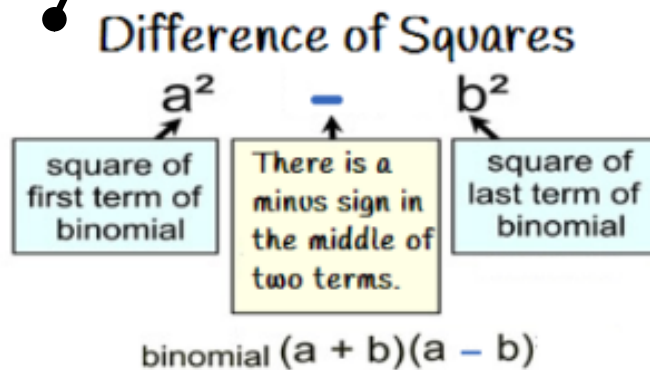
$$2y(y+3)^2$$

Factor the difference of squares.

21

$$16y^4 - 4y^2$$

The factored form of $16y^4 - 4y^2$ is .



$$= 4y^2(4y^2 - 1)$$
$$= 4y^2(2y + 1)(2y - 1)$$

22 Solve the following equation with special factors.

$$\sqrt{4x^2 - 4x + 1} = 0$$

$$(2x - 1)^2 = 0$$

Solve for x

$$\begin{array}{r} 2x - 1 = 0 \\ +1 \quad +1 \\ \hline \end{array}$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

Perfect Square Trinomial

$$a^2 \pm 2ab + b^2$$

square of first term of binomial

twice the product of binomial's first and last terms

square of last term of binomial

binomial $(a \pm b)^2$



The value of x is

.

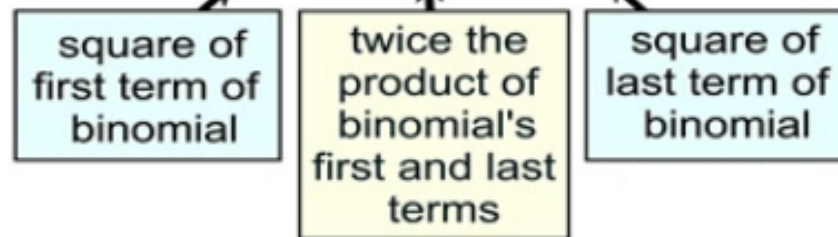
23 Factor the perfect-square trinomial.

$$\sqrt{36x^2 + 36x + 9}$$

$$(6x + 3)^2$$

Perfect Square Trinomial

$$a^2 \pm 2ab + b^2$$

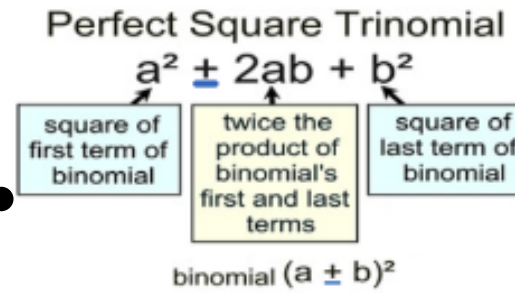


binomial $(a \pm b)^2$

Solve the following equation with special factors.

24

$$\frac{7x^4}{7x^2} + \frac{14x^3}{7x^2} + \frac{7x^2}{7x^2} = 0$$



$$7x^2(x^2 + 2x + 1) = 0$$

$$7x^2(x+1)^2 = 0$$

$$7x^2 = 0$$

$$x = 0$$

$$x + 1 = 0$$

$$\frac{-1}{-1} = -1$$

$$x = -1$$

The values of x are

or .



25

Solve the following equation with special factors.

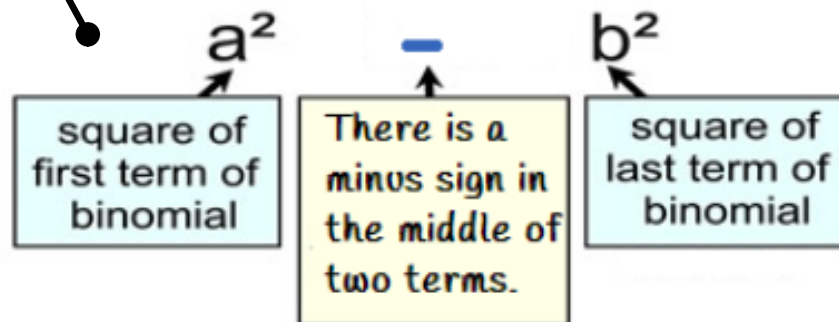
$$x^2 - 49 = 0 \quad (x - 7)(x + 7) = 0$$



The values of x that satisfy the equation are

or

Difference of Squares



binomial $(a + b)(a - b)$